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A MATHEMATICAL MODEL ON INDUSTRIALIZATION DUE TO HUMAN POPULATION

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Abstract:

In this paper, a nonlinear mathematical model is proposed and analyzed to study the Industrialization due to human population. We consider the variables namely, density of the human population and the Industries. We consider that the growth rate of industries is dependent on the density of human population. It is noted that for sustained industrialization, control measures on its growth are required to maintain the ecological stability. We discretize the model by applying Backward Euler method and analyse the stability of the model. Finally, we provide some numerical simulations using MATLAB.

Key Words: Backward Euler Method, Difference Equations, Logistic Growth & Human Population

1. Introduction:

As a city expands, industrial companies that were originally located outside the city perimeters find themselves surrounded by residential areas. Industries are necessary for the well being of the people of every country, because industries together with agriculture helps the country in achieving its economic growth and development. Industrialization has many positive effects on the society. The creation of power machines and factories provided many job opportunities. The machinery increased production speed of the goods. Industrialization also lead to urbanization. Urbanization is the movement of the people into cities. People wanted to live closer to the factories that they work. Industrialization increased agricultural and manufacturing output, allowing people to take jobs in other sectors and increasing the amount of consumer goods and food available to the population.

Large industries need thousands of skilled workers. It provides massive employment opportunity for a large population. Industrialization created significant population growth, as well as increased the economic output. It also enchanced technological development, enabling scientific advances that changed the world. Before the Industrial Revolution, most of the people were farmers, and most of the economic activity centered around small towns and villages. This left the communities isolated from one another even when the distances between them were relatively small. Industrialization concentrated populations in the cities, which soon became dependent on rural communities for food. The rural communities, in turn, became dependent on the cities for manufactured goods and tools to make their lives easier. There are also negative effects due to industrialization. Due to industrialization, there is a constant depletion of natural resources. Many industries are powered by thermal power plants that consumes coal. Since, large industries are spread over many acres of land, agricultural lands and forests are often cleared to make available for the required land. Global warming is one of the negative effects. Urban areas doubled, tripled or quadrupled in size led to overcrowding in cities. Sometimes a large population is a good thing, but in this case, the population is too big and causes many health problems. Living conditions are dirty and unhealthy. The lack of sanitation gets many people sick. Working conditions in the industries also hurts people by causing fatigue and illnesses [8], [9], [10]. For the Census of India in 2011, the definition of urban area is as follows:

- ✓ All places with a municipality, corporation, cantonment board or notified town area committee, etc. All other places which satisfied the following criteria:
- ✓ A minimum population of \$5,000\$.
- ✓ At least 75 percent of the male main working population engaged in non-agricultural pursuits.
- ✓ A density of the population of at least 400 persons per sq. km.

The policies given by the government of India to the industries are namely, The Industries (Development and Regulation) Act 1951 and The Industries (Development and Regulation) Amendment Act 2016.

B.Dubey, S. Sharma, P. Sinha, and J.B. Shukla analyzed the depletion of forestry resources by population and population pressure augmented industrialization [5]. Modelling the Depletion of Forestry Resources due to Crowding by Industrialization was studied by Vivi Ramdhani, Jaharuddin and Nugrahani. E. H [3]. In this paper, we construct a mathematical model on Industrialization due to human population. In section 2, a mathematical model is proposed and discretized. In section 3, we list the equilibrium points of the model. We analyze the stability of the model in section 4. Section 5 consists of Numerical simulations through MATLAB. Discussion is given in section 6.

2. The Mathematical Model:

The Mathematical model of Industrialisation due to human population is given below

$$\frac{dP}{dt} = r(1-\frac{P}{L})P - r_1 PI + (r_0 - r_2)P \qquad (1)$$
$$\frac{dI}{dt} = QP - Q_0 I$$

Where

P - Density of the Human population.
I - Density of the Industries.
r - Intrinsic growth rate of the human Population.

r_0 - Growth rate of Human population due to the Industries.

r_1 - Decline in growth rate of human population due to Industries.

r_2 - Natural death rate of Human population.

Q - Growth-rate of Industries due to human population.

Q_0 - Control rate of Industrialisation due to measures applied by government agencies.

K - Carrying Capacity of the human population.

Applying the Backward Euler Scheme to the system of equations (1), we obtain the discrete time model:

$$P_{n+1} = P_n + rP_{n+1}(1-\frac{P_{n+1}}{K}) - r_1 P_{n+1} I_{n+1} + (r_0 - r_2)P_{n+1} \qquad (2)$$
$$I_{n+1} = I_n + QP_{n+1} - Q_0 I_{n+1}$$

3. Equilibrium Points:

i. $E_1 = (\bar{P}, 0)$

Where $\bar{P} = \dfrac{K(r + r_0 - r_2)}{r}$

and $r + r_0 > r_2$ (3)

ii. $E_2 = (P^*, I^*)$

Where $P^* = (r + r_0 - r_2)\left[\dfrac{KQ_0}{rQ_0 + KQr_1}\right], I^* = (r + r_0 - r_2)\left[\dfrac{KQ}{rQ_0 + KQr_1}\right]$

and $r + r_0 > r_2$. (4)

The jacobian matrix of the system (2) is given by

$$J = \begin{pmatrix} 1 + r - \frac{2rP}{K} - r_1 I + (r_0 - r_2) & -r_1 P \\ Q & 1 - Q_0 \end{pmatrix} \qquad (5)$$

4. Stability of the Mathematical Model:

Theorem 1:

The fixed point $E_1 = (\bar{P}, 0)$ is stable if $\dfrac{r(2-Q_0)[(r+r_0-r_2)-2]}{Kr_1(r+r_0-r_2)} < Q < \dfrac{r\{(r-r_0+r_2)(1-Q_0)+Q_0\}}{Kr_1(r+r_0-r_2)}$

Otherwise unstable.

Proof:

Consider the jacobian matrix of the system (2) with respect to the fixed point E_1.

$$J_1 = \begin{pmatrix} 1 + r - \frac{2r\bar{P}}{K} + (r_0 - r_2) & -r_1 \bar{P} \\ Q & 1 - Q_0 \end{pmatrix} \qquad (6)$$

The characteristic equation of the above matrix is given by,

$$\phi(\lambda) = \lambda^2 - (1-(r+r_0-r_2))\lambda - (1-Q_0)\lambda + (1-(r+r_0-r_2))(1-Q_0) + \frac{Kr_1 Q}{r}(r+r_0-r_2) = 0$$

It follows from the well-known Jury conditions that the modulus of all the roots of the above characteristic equation is less than 1 if and only if the conditions $\phi(1) > 0, \phi(-1) > 0$ and $|\det J_1| < 1$ hold[2].

i) $\phi(1) > 0 \Rightarrow rQ_0 + Kr_1 Q > 0$

ii) $\phi(-1) > 0 \Rightarrow Q > \dfrac{r(2-Q_0)[(r+r_0-r_2)-2]}{Kr_1(r+r_0-r_2)}$

iii) $|\det J_1| < 1 \Rightarrow Q < \dfrac{r\{(r-r_0+r_2)(1-Q_0)+Q_0\}}{Kr_1(r+r_0-r_2)}$

Using the above conditions and (4), we can say that the fixed point E_1 is stable if

$$\frac{r(2-Q_0)[(r+r_0-r_2)-2]}{Kr_1(r+r_0-r_2)} < Q < \frac{r\{(r-r_0+r_2)(1-Q_0)+Q_0\}}{Kr_1(r+r_0-r_2)}$$

Theorem 2:

The fixed point $E_2 = (P^*, I^*)$ is stable if $\dfrac{rQ_0(2-Q_0)[(r+r_0-r_2)-2]}{Kr_1[2(2-Q_0)+Q_0(r+r_0-r_2)]} < Q < \dfrac{rQ_0}{Kr_1}\left\{\dfrac{1-Q_0(r+r_0-r_2-1)}{Q_0(r+r_0-r_2-1)}\right\}$

Otherwise unstable.

Proof:

Consider the Jacobean matrix of the system (2) with respect to the fixed point E_2.

$$J_2 = \begin{pmatrix} 1+r-\dfrac{2rP^*}{K}-r_1I^*+(r_0-r_2) & -r_1P^* \\ Q & 1-Q_0 \end{pmatrix} \tag{7}$$

The characteristic equation of the above matrix is given by,

$$\varphi(\lambda) = \lambda^2 - \Omega_1\lambda + \Omega_2 = 0$$

Where $\Omega_1 = \left[1-\dfrac{rQ_0(r+r_0-r_2)}{rQ_0+r_1KQ}\right]+[1-Q_0]$

$$\Omega_2 = \left[1-\frac{rQ_0(r-r_2+r_0)}{rQ_0+r_1KQ}\right]\times[1-Q_0]+\frac{Kr_1QQ_0(r+r_0-r_2)}{rQ_0+r_1KQ}$$

It follows from the well-known Jury conditions that the modulus of all the roots of the above characteristic equation is less than 1 if and only if the conditions $\varphi(1)>0, \varphi(-1)>0$ and $|\det J_2|<1$ hold [2].

i) $\varphi(1)>0 \Rightarrow rQ_0 + Kr_1Q > 0$

ii) $\varphi(-1)>0 \Rightarrow Q > \dfrac{rQ_0(2-Q_0)[(r+r_0-r_2)-2]}{Kr_1[2(2-Q_0)+Q_0(r+r_0-r_2)]}$

iii) $|\det J_2|<1 \Rightarrow Q < \dfrac{rQ_0}{Kr_1}\left\{\dfrac{1-Q_0(r+r_0-r_2-1)}{Q_0(r+r_0-r_2-1)}\right\}$

Using the above conditions, (4) and $Q_0 < 2$, we can say that the fixed point E_2 is stable if

$$\frac{rQ_0(2-Q_0)[(r+r_0-r_2)-2]}{Kr_1[2(2-Q_0)+Q_0(r+r_0-r_2)]} < Q < \frac{rQ_0}{Kr_1}\left\{\frac{1-Q_0(r+r_0-r_2-1)}{Q_0(r+r_0-r_2-1)}\right\}$$

5. Numerical Simulations:

Numerical simulations have been carried out to investigate the dynamics of the proposed model. The simulations have been performed using MATLAB.

Taking the following set of parametric values:

$$r = 1.02, r_0 = 0.5, r_1 = 0.2, r_2 = 1.1, Q = 0.3, K = 5$$

We assume different values of Q_0 to investigate the control measures given by the government and it is shown by figure 1, figure 2 and figure 3.

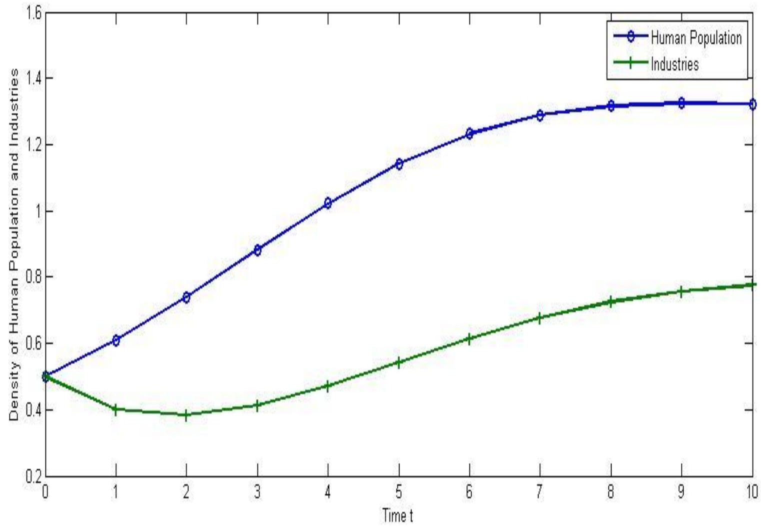

Figure 1: Dynamics of the system with $Q_0 = 0.5$

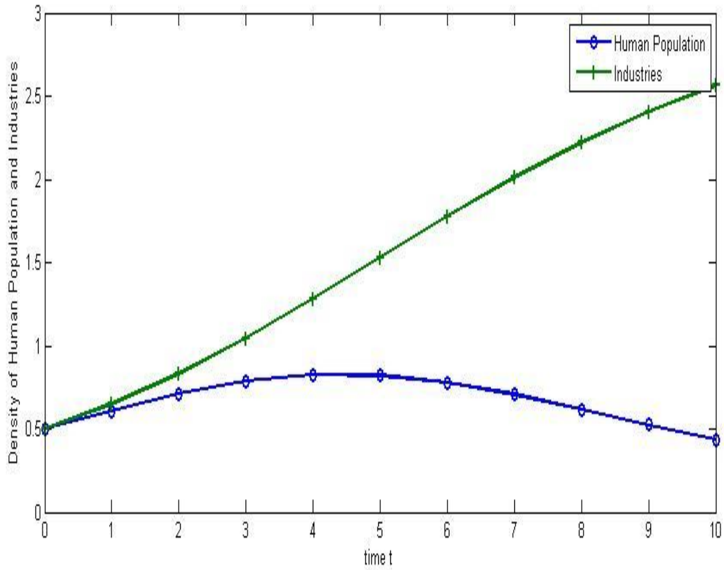

Figure 2: Dynamics of the system with $Q_0 = 0$

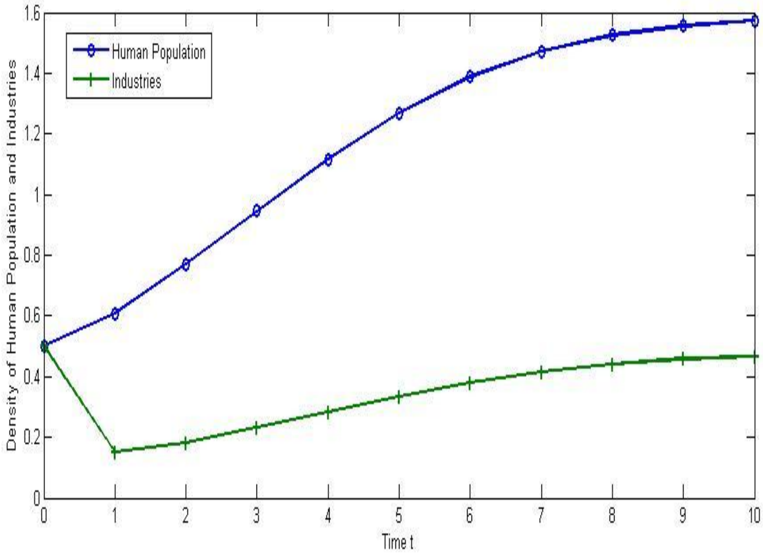

Figure 3: Dynamics of the system with $Q_0 = 1$

6. Discussion:

We have proposed a Discrete-time model on Industrialization due to human population. We consider different values for is coefficient of control rate for industrialization applied by government. And we can see that when $Q = 0.3, Q_0 = 0$ the density of industries increases as a result density of human population decreases. And when $Q = 0.3, Q_0 = 1$ the density of industries decreases while the density of human population is increased.

7. Conclusion:

In this paper, we have considered a discrete-time model on industrialization due to human population. We have considered the growth rate of human population to be logistic. And the growth rate of industries depend on density of human population. We list the equilibrium points of the model and analyze the stability around each equilibrium point. We have proved our theoretical results using numerical simulations through MATLAB. Finally we have discussed different values for the coefficient of control rate for industrialization applied by government in the numerical simulations.

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