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ON SOLVING FULLY FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEM BY USING SYMMETRIC TRAPEZOIDAL FUZZY NUMBERS

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Abstract:

In this paper, a new approach for solving fully fuzzy multi-objective linear programming problem (FFMLPP) by using symmetric trapezoidal fuzzy numbers. Fuzzy arithmetic operations and simplex algorithm is proposed for solving fully fuzzy multi-objective linear programming problem with parameters represented by symmetric trapezoidal fuzzy numbers without converting it to crisp numbers. Using the proposed method the fuzzy optimal solution of fully fuzzy multi-objective linear programming problem can be easily obtained. Finally a numerical example is provided to check the efficiency of the proposed method.

Key Words: Fully Fuzzy Number, Ranking Function & Symmetric Trapezoidal Fuzzy Numbers

1. Introduction:

Most of the real life problems exhibit properties of multi-objectivity and fuzziness in nature. Bellman and Zadeh [2] proposed the concepts of decision making in fuzzy environments. The fuzzy linear programming problem in which all parameters and variables are represented by fuzzy numbers in known as fully fuzzy linear programming problems. Hosseinzadeh Lotfi [7] (2009) discussed fully fuzzy multi-objective linear programming problem where all variables are symmetric triangular fuzzy numbers. In this paper, a new method by using Ezzati et al [4] (2014)'s fuzzy arithmetic operations and fuzzy version of simplex algorithm is proposed for solving fully fuzzy multi-objective linear programming problem by using symmetric trapezoidal fuzzy numbers. In section 2 some basic definitions and arithmetic operations of symmetric trapezoidal fuzzy numbers. In section 3 a new method is proposed for fully fuzzy multi-objective linear programming problem and in section 4 a numerical example is solved. Finally conclusions are drawn in section 5.

2. Preliminaries:

In this section, some basic definitions and arithmetic operations of symmetric trapezoidal fuzzy numbers and ranking function.

Definition 2.1 [11]: Let X be a universal set. Then fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}} : X \to [0,1]$

Which assign a real number $\mu_{\tilde{A}(x)}$ at x shows the grade of membership x in $\tilde{A}$.

Definition 2.2 [11]: A fuzzy number $\tilde{A} = (a^L, a^U, \alpha, \alpha)$ is said to be a symmetric trapezoidal fuzzy number if its membership function is given as follows,

$$\mu_{\tilde{A}'}(x) = \begin{cases} \dfrac{x-(a^L-\alpha)}{\alpha} & a^L-\alpha \le x \le a^L \\ 1 & a^L \le x \le a^U \\ \dfrac{(a^U+\alpha)-x}{\alpha} & a^U \le x \le a^U+\alpha \\ 0 & \end{cases}$$

Definition 2.3 [11]: Let $\tilde{A} = (a^L, a^U, \alpha, \alpha)$ and $\tilde{B} = (b^L, b^U, \beta, \beta)$ be two trapezoidal fuzzy numbers then

$$\tilde{A} + \tilde{B} = (a^L + b^L, a^U + b^U, \alpha + \beta, \alpha + \beta)$$

(i) $$\tilde{A} - \tilde{B} = (a^L - b^U, a^U - b^L, \alpha + \beta, \alpha + \beta)$$

$$\tilde{A} \times \tilde{B} = \left(\frac{a^L + a^U}{2} \times \frac{b^L + b^U}{2} - W, \frac{a^L + a^U}{2} \times \frac{b^L + b^U}{2} + W, \left|W - W'\right|, \left|W - W\right|\right)$$

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Where $w = \min(\left[\frac{a^L + a^U}{2}\right]\left[\frac{b^L + b^U}{2}\right] - \min(m), \max(m) - \left[\frac{a^L + a^U}{2}\right]\left[\frac{b^L + b^U}{2}\right])$

$$w' = \min(\left[\frac{a^L + a^U}{2}\right]\left[\frac{b^L + b^U}{2}\right] - \min(n), \max(n) - \left[\frac{a^L + a^U}{2}\right]\left[\frac{b^L + b^U}{2}\right])$$

and $m = (a^L b^L, a^L b^U, a^U b^L, a^U b^U)$

$$n = ((a^L - \alpha)(b^L - \beta), (a^L - \alpha)(b^U + \beta), (a^L + \alpha)(b^L - \beta), (a^L + \alpha)(b^U + \beta))$$

$$\tilde{A} \div \tilde{B} = (\frac{a^L + a^U}{b^L + b^U} - W, \frac{a^L + a^U}{b^L + b^U} + W, \left|W - W'\right|, \left|W - W\right|)$$

$$w = \min(\left[\frac{a^L + a^U}{b^L + b^U}\right] - \min(m), \max(m) - \left[\frac{a^L + a^U}{b^L + b^U}\right])$$

$$w' = \min(\left[\frac{a^L + a^U}{b^L + b^U}\right] - \min(n), \max(n) - \left[\frac{a^L + a^U}{b^L + b^U}\right])$$

$$m = (\frac{a^L}{b^L}, \frac{a^L}{b^U}, \frac{a^U}{b^L}, \frac{a^U}{b^U}), n = (\frac{a^L - \alpha}{b^L - \beta}, \frac{a^L - \alpha}{b^U + \beta}, \frac{a^U + \alpha}{b^L - \beta}, \frac{a^U + \alpha}{b^U + \beta})$$

Definition 2.4 [11]: A ranking function is a function R: F(X) →X which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{A} = (a^L, a^U, \alpha, \alpha)$ be two symmetric trapezoidal fuzzy numbers then

$$R(\tilde{A}) = \frac{a^L + a^U}{2}$$

Definition 2.5 [11]: Let $\tilde{A} = (a^L, a^U, \alpha, \alpha)$ and $\tilde{B} = (b^L, b^U, \beta, \beta)$ be two trapezoidal fuzzy numbers then

$$\tilde{A} \le \tilde{B} \text{ iff } R(\tilde{A}) \le R(\tilde{B})$$

$$\tilde{A} < \tilde{B} \text{ iff } R(\tilde{A}) < R(\tilde{B})$$

3. Proposed Method:

In this section, the steps of the fuzzy simplex algorithm are proposed to obtain the fuzzy optimal solution for fully fuzzy multi-objective linear programming problem as follows:

Step1: convert the all the inequalities of the constraints into equation by introducing fuzzy slack or surplus variables.

Step2: construct the fuzzy simplex table

Step3: find $\tilde{z}_j - \tilde{c}_j = \sum_{i=1}^{m} \tilde{c}_{B_i} \tilde{a}_{ij} - \tilde{c}_j$

If $\tilde{z}_j - \tilde{c}_j \ge 0$, then the fuzzy basic feasible solution, obtained by using values of $\tilde{x}_{B_1}, \ldots \tilde{x}_{B_m}$ is fuzzy optimal solution and $\tilde{Z}$ is the optimal value of maximization problem.If there exists any j such that $\tilde{z}_j - \tilde{c}_j \le 0$, processed the next step.

Step 4: if there exists one or more fuzzy variables with $\tilde{z}_j - \tilde{c}_j \le 0$, then the corresponding to which rank of $\tilde{z}_j - \tilde{c}_j$ is most negative will enter the basis. Let it be $\tilde{z}_r - \tilde{c}_r$ for some j=r

If $\tilde{a}_{ij} \le 0, \forall i$ then there exists fuzzy unbounded solution to the given FFMLP problem.

If $\tilde{a}_{ij} \ge 0$, for one or more values of I then compute $\min R\left(\frac{\tilde{l}_i}{\tilde{a}_{ir}}\right), \tilde{a}_{ir} \succ \tilde{0}, i = 1,2 \ldots m$

Where $\tilde{l}_i$ is the value of i-th fuzzy basic variable. The fuzzy variable, corresponding to which minimum occurs, will leave the basis. Let the minimum occurs corresponding to $\tilde{x}_{B_k}$ then the common element $\tilde{a}_{kr} \ge 0$, Which occurs at intersection of k-th row and r-th column is known as the leading element.

Step 5: construct the new fuzzy simplex table and calculate the new entries for the fuzzy simplex table as follows

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$\hat{a}_{kj} = \frac{\tilde{a}_{kj}}{\tilde{a}_{kr}}$ and $\hat{a}_{ij} = \hat{a}_{ij} - \frac{\tilde{a}_{kj}}{\tilde{a}_{kr}}\tilde{a}_{ir}, i = 1,2,3....m, i \neq k, j = 1,2...., n$

Step 6: Repeat the computational procedure from step3 to step5 until either (i) step 3 or(i)of step 4 is satisfied.

Step 7:if $\tilde{z}_j - \tilde{c}_j = \tilde{0}$ corresponding to any fuzzy non basic variable in fuzzy optimal table then a fuzzy alternative solution may exist and to find it enter that fuzzy non basic variable into the basis and repeat once step3 .

Step 8: The optimal solution is obtained by preemptive optimization method by considering the other objective functions one by one as per ranking.

4. Numerical Example:

Consider the following problem

$$\max\ \tilde{z} = (3,5,2,2)\tilde{x}_1 + (2,18,4,4)\tilde{x}_2$$
$$\max\ \tilde{z} = (2,2,3,3,)\tilde{x}_1 + (2,4,4,4)\tilde{x}_2$$

Subject to

$$(1,3,2,2)\tilde{x}_1 + (2,18,4,4)\tilde{x}_2 \leq (48,52,4,4)$$
$$(2,2,5,5)\tilde{x}_1 + (2,8,4,4)\tilde{x}_2 \leq (20,180,6,6)$$
$$(1,3,4,4)\tilde{x}_1 + (2,4,5,5)\tilde{x}_2 \leq (30,150,20,20)$$

The problem can be written as

$$\max\ \tilde{z} = (3,5,2,2)\tilde{x}_1 + (2,18,4,4)\tilde{x}_2$$
$$\max\ \tilde{z} = (2,2,3,3,)\tilde{x}_1 + (2,4,4,4)\tilde{x}_2$$

Subject to

$$(1,3,2,2)\tilde{x}_1 + (2,18,4,4)\tilde{x}_2 + (1,10,0)\tilde{x}_3 \approx (48,52,4,4)$$
$$(2,2,5,5)\tilde{x}_1 + (2,8,4,4)\tilde{x}_2 + (1,10,0)\tilde{x}_4 \approx (20,180,6,6)$$
$$(1,3,4,4)\tilde{x}_1 + (2,4,5,5)\tilde{x}_2 + (1,1,0,0)\tilde{x}_5 \approx (30,150,20,20)$$
$$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5 \geq \tilde{0}$$

Table 1

X_B	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	RHS
$\tilde{z}$	(-5,-3,2,2)	(-18,-2,4,4)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{x}_3$	(1,3,2,2)	(1,1,2,2)	(1,1,0,0)	$\tilde{0}$	$\tilde{0}$	(48,52,4,4)
$\tilde{x}_4$	(2,2,5,5)	(2,8,4,4)	$\tilde{0}$	(1,1,0,0)	$\tilde{0}$	(20,180,6,6)
$\tilde{x}_5$	(1,3,4,4)	(2,4,5,5)	$\tilde{0}$	$\tilde{0}$	(1,1,0,0)	(30,150,2,2)

The proposed algorithm, fuzzy variable $\tilde{x}_2$,will enter the basis and the fuzzy variable $\tilde{x}_4$ will leave the basis.

X_B	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	RHS
$\tilde{z}$	(-1/5,5/5,17/10,17/10)	$\tilde{0}$	$\tilde{0}$	(2,2,1,1)	$\tilde{0}$	(120,280,30,30)
$\tilde{x}_3$	(-1/5,5/5,17/10,17/10)	$\tilde{0}$	(1,1,0,0)	(-1/5,-1/5,3,3)	$\tilde{0}$	(20,40,4,4)
$\tilde{x}_2$	(-2/5,3/5,5/10,5/10)	(1,1,0,0)	$\tilde{0}$	(1/5,1/5,2,2)	$\tilde{0}$	(20,20,6,6)
$\tilde{x}_5$	(-6/5,8/5,9/10,9/10)	$\tilde{0}$	$\tilde{0}$	(-3/5,-3/5,3,3)	(1,1,0,0)	(30,30,2,2)

The next objective function is

$$\max\ \tilde{z} = (2,2,3,3,)\tilde{x}_1 + (2,4,4,4)\tilde{x}_2$$

Subject to

$$(1,3,2,2)\tilde{x}_1 + (2,18,4,4)\tilde{x}_2 \leq (48,52,4,4)$$
$$(2,2,5,5)\tilde{x}_1 + (2,8,4,4)\tilde{x}_2 \leq (20,180,6,6)$$
$$(1,3,4,4)\tilde{x}_1 + (2,4,5,5)\tilde{x}_2 \leq (30,150,20,20)$$
$$(0,0,11)\tilde{x}_1 + (20,20,6,6)\tilde{x}_2 \leq (120,280,30,30)$$
$$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5, \tilde{x}_6 \geq \tilde{0}$$

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Table 2

X_B	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	$\tilde{x}_6$	RHS
$\tilde{z}$	(-2,-2,3,3)	(-4,-2,4,4)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{x}_3$	(1,3,2,2)	(1,1,2,2)	(1,1,0,0)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	(48,52,4,4)
$\tilde{x}_4$	(2,2,5,5)	(2,8,4,4)	$\tilde{0}$	(1,1,0,0)	$\tilde{0}$	$\tilde{0}$	(20,180,6,6)
$\tilde{x}_5$	(1,3,4,4)	(2,4,5,5)	$\tilde{0}$	$\tilde{0}$	(1,1,0,0)	$\tilde{0}$	(30,150,2,2)
$\tilde{x}_6$	$\tilde{0}$	(20,20,6,6)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	(1,1,0,0)	(120,280,30,30)

.

X_B	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	$\tilde{x}_6$	RHS
$\tilde{z}$	(-2,-2,3,3)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	(0.12,.0.18,3,3)	(20,40,4,4)
$\tilde{x}_3$	(1,3,2,2)	$\tilde{0}$	(1,1,0,0)	$\tilde{0}$	$\tilde{0}$	(-0.05,-0.05,2,2)	(38,42,4,4)
$\tilde{x}_4$	(2,2,5,5)	$\tilde{0}$	$\tilde{0}$	(1,1,0,0)	$\tilde{0}$	(-0.30,-0.20,4,4)	(48,52,4,4)
$\tilde{x}_5$	(1,3,4,4)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	(1,1,0,0)	(-0.12,.-0.18,3,3)	(42,118,2,2)
$\tilde{x}_2$	$\tilde{0}$	(1,1,0,0)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	(0.07,0.03,1,1)	(10,10,3,3)

.

X_B	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	$\tilde{x}_6$	RHS
$\tilde{z}$	$\tilde{0}$	$\tilde{0}$	(1,1,0,0)	$\tilde{0}$	$\tilde{0}$	(0.12,.0.08,3,3)	(60,80,4,4)
$\tilde{x}_1$	(1,1,0,0)	$\tilde{0}$	(0.50,0.50,1,1)	$\tilde{0}$	$\tilde{0}$	(-0.03,-0.03,2,2)	(20,20,6,6)
$\tilde{x}_4$	$\tilde{0}$	$\tilde{0}$	(-1,-1,1,1)	(1,1,0,0)	$\tilde{0}$	(-0.10,-0.30,4,4)	(10,10,3,3)
$\tilde{x}_5$	$\tilde{0}$	$\tilde{0}$	(-1,-1,1,1)	$\tilde{0}$	(1,1,0,0)	(-0.12,.-0.08,3,3)	(20,20,6,6)
$\tilde{x}_2$	$\tilde{0}$	(1,1,0,0)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	(0.07,0.03,1,1)	(10,10,3,3)

The fuzzy optimal solution of the for fully fuzzy multi-objective linear programming problem is $\tilde{x}_1 = (20,20,6,6)$ $\tilde{x}_2 = (10,10,3,3)$ and the optimal value is $\tilde{z} = (60, 80, 4, 4)$

5. Conclusion:

A new method by using Ezzati et al(2014)'s fuzzy arithmetic operation and a fuzzy version of simplex algorithm is proposed for solving fully fuzzy multi objective linear programming problem whose parameters are symmetric trapezoidal fuzzy number without converting the given problem into crisp equivalent problem. The proposed method can be extended to fully intuitionstic fuzzy multi objective linear programming problem in near future

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