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A STUDY ON ROUGH FUZZY BI-QUASI-IDEALS OF Γ-SEMIGROUPS

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Abstract:

In this paper basic notions of rough fuzzy bi-ideals and rough fuzzy quasi-ideals are given and we discuss some of its basic properties. We introduce the notion of rough fuzzy bi-quasi-ideals of Γ-semigroups as a generalization of fuzzy bi-quasi-ideals and we characterize regular Γ-semigroups interms of rough fuzzy bi-quasi-ideals of Γ-semigroups.

Index Terms: Γ-semigroups, rough fuzzy quasi-ideals, rough fuzzy bi-quasi-ideals

1. Introduction:

The fundamental concept of Γ-semigroup was introduced by Sen [17]. Many researchers have worked on Γ-semigroup and its sub structures. Many Classical notion of semigroups have be extended to Γ-semigroups by Sha and Sen [15,16]. R. Chinram and C. Jirojkul [6] Studied bi-ideals in Γ-semigroups. In some sense almost all concepts we are meeting in everyday life, are vague rather than precise. On the contrary, it is interesting to see that classical mathematics requires that all mathematical notions must be exact, otherwise precise reasoning would be impossible. To reduce this gap between the real world full of vagueness and the traditional mathematics purely concern precise concepts, some kind of theories were given like theory of fuzzy sets, rough sets etc. The concept of fuzzy set was introduced by L.A.Zadeh [22] in 1965 and studied their properties on the parallel line to set theory. The fuzzification of algebraic structures was introduced by Rosenfeld [14]. Rosenfeld defined the fuzzy subgroup and gave some of its properties. W.J.Liu[8] introduced fuzzy ideals in ring. The notion of rough set was introduced by Z.Pawlak[9-12] in 1982. Some authors for example [3,5,13,21] have studied algebraic properties of rough set. Biswas and Nanda [2], introduced the notion of rough subgroups. Thillaigovindan et.al., [20] stuidied rough ideals in Γ-semigroups. V.S.Subha[18,19] studied rough k-ideals in semirings. There are many papers studying the connection and differences between fuzzy set theory [1,4]. Dubois and Parade [7] were one of the first to investigate the problems of fuzzification of rough sets. This paper as a generalization of rough ideals, we introduce the notion of rough fuzzy bi-quasi-ideals in Γ-semigroup. We characterize the regular Γ-semigroup interms of fuzzy bi-quasi-ideals of Γ-semigroup.

2. Preliminaries Notes:

Let θ be a congruence relation on M, that is θ is an equivalence relation on M such that $(a,b) \in \theta \Rightarrow (a\gamma x, b\gamma x) \in \theta$ and $(x\gamma a, x\gamma b) \in \theta$ for all $a,x,b \in \Gamma$. If θ is a congruence relation on M, then for every $x \in M$, $[x]_\theta$ denotes the congruence class of x with respect to the relation θ. A congruence relation θ on M is called complete if $[a]_\theta \Gamma [b]_\theta = [a\Gamma b]_\theta$ for every $a, b \in M$.

Let M be a nonempty subset Γ -semigroup. A mapping $\mu: M \to [0,1]$ is called a fuzzy subset of M. If μ is a fuzzy subset of M, for $t \in [0,1]$ then the set $\mu_t = \{x \in M \mid \mu(x) \geq t\}$ is called a level subset of M with respect to a fuzzy subset of μ, A fuzzy subset $\mu: M \to [0,1]$ is a non empty fuzzy subset if μ is not a constant function. For any two fuzzy subsets μ and λ of M, $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$ for all $x \in M$.

Let A be a non empty subset M. The characteristic function of M is defined by

$$\chi_A(x) = \begin{cases} 1, & if\ x \in A: \\ 0, & if\ x \notin A. \end{cases}$$

Let μ and γ be two fuzzy subsets of Γ-Semigroup M and $x, y, z \in M, \alpha \in \Gamma$. We define

$$\mu \circ \gamma(x) = \begin{cases} sup_{x=y\alpha z}, min\{\mu(y), \gamma(z)\}: \\ 0, \qquad otherwise. \end{cases}$$

$\mu \cap \gamma(x) = min\{\mu(x), \gamma(x)\}$, for all $x \in M$.

A fuzzy subset μ of Γ -semigroup M is called a

- a fuzzy Γ -subsemigroup of M if $\mu(x\alpha y) \geq \mu(x) \wedge \mu(y)$.
- a fuzzy left(right) ideal of M if $\mu(x\alpha y) \geq \mu(y)(\mu(x))$.
- a fuzzy ideal of M if $\mu(x\alpha y) \geq \mu(x) \vee \mu(y)$.
- a fuzzy left(right) ideal of M if $\chi_M \circ \mu \subseteq \mu (\mu \circ \chi_M \subseteq \mu)$.
- a fuzzy bi-ideal of M if $\mu \circ \chi_M \circ \mu \subseteq \mu$.
- a fuzzy quasi-ideal of M if $\mu \circ \chi_M \cap \chi_M \circ \mu \subseteq \mu$.

Definition 2.1: [8] A fuzzy subset μ of M is said to be fuzzy left (resp. right) bi-quasi-ideal of M if $\chi_M \circ \mu \cap \mu \circ \chi_M \circ \mu \subseteq \mu$ $(\mu \circ \chi_M \cap \mu \circ \chi_M \circ \mu \subseteq \mu)$. A fuzzy subset μ of a Γ -semigroup M is called a fuzzy bi-quasi-ideal if it is both a fuzzy left bi-quasi-ideal and fuzzy right bi-quasi-ideal.

Theorem 2.2: [8] Let M be a Γ -semigroup and μ be a non empty fuzzy subset of M. Then μ is a fuzzy bi-quasi-ideal of M if and only if the level subset μ_t of μ is a bi-quasi-ideal of M for every t ∈ [0,1].

Theorem 2.3: [8] Let I be a non empty subset of Γ -semigroup M and χ_I be the characteristic function of I. Then I is a bi-quasi-ideal of M if and only if χ_I is a fuzzy bi-quasi-ideal of M.

Theorem 2.4: [5] Let θ be a congruence relation on M. If μ is a fuzzy left ideal of M. Then,

- μ is a θ-upper rough fuzzy ideal of M.
- If θ is complete and $\underline{\theta}(\mu) \neq \emptyset$ then μ is a θ-lower rough fuzzy ideal of M.

3. Main Results:

In this section, as a generalization of rough fuzzy ideals, we introduce the notion of rough fuzzy bi-quasi-ideal *Γ*-semigroup and study the properties of rough fuzzy bi-quasi ideals.

Theorem 3.1:

Let θ be a congruence relation on M. If μ is a fuzzy left ideal of M . Then,

- μ is a θ-upper rough fuzzy bi-quasi-ideal of M.
- If θ is complete and $\underline{\theta}(\mu) \neq \emptyset$ then μ is a θ-lower rough fuzzy bi-quasi-ideal of M.

Proof:

(i) Let μ is a fuzzy left ideal of M. Then $\overline{\theta}(\mu)$ is a fuzzy left ideal of M and $x \in M, \alpha \in \Gamma$. We have

$$\begin{aligned}\chi_M \circ \overline{\theta}(\mu)(x) &= sup_{x=y\alpha z}\left\{min\{\chi_M(y), \overline{\theta}(\mu)(z)\}\right\}\\ &= sup_{x=y\alpha z}\left\{min\{1, \overline{\theta}(\mu)(z)\}\right\}\\ &= sup_{x=y\alpha z}\{\overline{\theta}(\mu)(z)\}\\ &\leq sup_{x=y\alpha z}\{\overline{\theta}(\mu)(y\alpha z)\}\\ &= sup_{x=y\alpha z}\{\vee_{a\in[y\alpha z]_\theta}\mu(a)\}\\ &= \vee_{a\in[y\alpha z]_\theta}\mu(a)\\ &= \vee_{a\in[x]_\theta}\mu(a)\\ &= \overline{\theta}(\mu)(x).\end{aligned}$$

This implies that $\chi_M \circ \overline{\theta}(\mu)(x) \leq \overline{\theta}(\mu)(x)$.--------------(1)

Now $\overline{\theta}(\mu)\circ\chi_M \circ\overline{\theta}(\mu)(x) = sup_{x=a\alpha b\beta c}\left\{min\{\overline{\theta}(\mu)(a), \chi_M \circ\overline{\theta}(\mu)(b\beta c)\}\right\}$

$$\begin{aligned}&\leq sup_{x=a\alpha b\beta c}\left\{min\{\overline{\theta}(\mu)(a), \overline{\theta}(\mu)(b\beta c)\}\right\}\\ &= \overline{\theta}(\mu)(x)\end{aligned}$$

This implies that $\overline{\theta}(\mu)\circ\chi_M \circ\overline{\theta}(\mu)(x) \leq \overline{\theta}(\mu)(x)$.--------------(2)

From (1) and (2) we have

$$\chi_M\circ\overline{\theta}(\mu) \cap \overline{\theta}(\mu)\circ\chi_M \circ\overline{\theta}(\mu)(x) \leq (\overline{\theta}(\mu) \cap \overline{\theta}(\mu))(x) = \overline{\theta}(\mu)(x)$$

Then $\chi_M\circ\overline{\theta}(\mu) \cap \overline{\theta}(\mu)\circ\chi_M \circ\overline{\theta}(\mu) \subseteq \overline{\theta}(\mu)$.

Similarly we prove $\overline{\theta}(\mu)\circ\chi_M \cap \overline{\theta}(\mu)\circ\chi_M \circ\overline{\theta}(\mu) \subseteq \overline{\theta}(\mu)$.

We obtain $\overline{\theta}(\mu)$ is fuzzy bi-quasi-ideal *of M*.

Therefore μ is a θ-upper rough fuzzy bi-quasi-ideal of M.

(ii) Let μ is a fuzzy left ideal of M. Then $\underline{\theta}(\mu)$ is a fuzzy left ideal of M and $x \in M$, $\alpha \in \Gamma$. We have

$$\begin{aligned}\chi_M \circ \underline{\theta}(\mu)(x) &= sup_{x=y\alpha z}\left\{min\{\chi_M(y), \underline{\theta}(\mu)(z)\}\right\}\\ &= sup_{x=y\alpha z}\left\{min\{1, \underline{\theta}(\mu)(z)\}\right\}\\ &= sup_{x=y\alpha z}\{\underline{\theta}(\mu)(z)\}\\ &\leq sup_{x=y\alpha z}\{\underline{\theta}(\mu)(y\alpha z)\}\\ &= sup_{x=y\alpha z}\{\wedge_{a\in[y\alpha z]_\theta}\mu(a)\}\\ &= \wedge_{a\in[y\alpha z]_\theta}\mu(a)\\ &= \wedge_{a\in[x]_\theta}\mu(a)\\ &= \underline{\theta}(\mu)(x).\end{aligned}$$

This implies that $\chi_M \circ\underline{\theta}(\mu)(x) \leq \underline{\theta}(\mu)(x)$.------------------(3)

Now $\underline{\theta}(\mu)\circ\chi_M \circ\underline{\theta}(\mu)(x) = sup_{x=a\alpha b\beta c}\left\{min\{\underline{\theta}(\mu)(a), \chi_M \circ\underline{\theta}(\mu)(b\beta c)\}\right\}$

$$\leq sup_{x=a\alpha b\beta c}\left\{min\{\underline{\theta}(\mu)(a),\underline{\theta}(\mu)(b\beta c)\}\right\}$$
$$=\underline{\theta}(\mu)(x)$$
$$\underline{\theta}(\mu)\circ\chi_M\circ\underline{\theta}(\mu)(x)\leq\underline{\theta}(\mu)(x).\text{------------------(4)}$$

From (3) and (4) we have

$$\chi_M\circ\underline{\theta}(\mu)\cap\underline{\theta}(\mu)\circ\chi_M\circ\underline{\theta}(\mu)(x)\leq\underline{\theta}(\mu)(x)\wedge\underline{\theta}(\mu)(x)=\underline{\theta}(\mu)(x)$$

Then $\chi_M\circ\underline{\theta}(\mu)\cap\underline{\theta}(\mu)\circ\chi_M\circ\underline{\theta}(\mu)\subseteq\underline{\theta}(\mu)$.
Similarly we prove $\underline{\theta}(\mu)\circ\chi_M\cap\underline{\theta}(\mu)\circ\chi_M\circ\underline{\theta}(\mu)\subseteq\underline{\theta}(\mu)$.
We obtain $\underline{\theta}(\mu)$ is fuzzy bi-quasi-ideal of M.
Therefore μ is a θ-lower rough fuzzy bi-quasi-ideal of M.

Theorem 3.2:

Let θ be a congruence relation on M. If μ is a fuzzy right ideal of M . Then,

- μ is a θ-upper rough fuzzy bi-quasi-ideal of M.
- If θ is complete and $\underline{\theta}$ (μ) ≠ ∅ then μ is a θ-lower rough fuzzy bi-quasi-ideal of M.

Proof:

(i) Let μ is a fuzzy right ideal of M. Then $\overline{\theta}$(μ) is a fuzzy right ideal of M and $x\in M,\alpha\in\Gamma$. We have

$$\overline{\theta}(\mu)(x)\circ\chi_M=sup_{x=y\alpha z}\left\{min\{\overline{\theta}(\mu)(y),\chi_M(z)\}\right\}$$
$$=sup_{x=y\alpha z}\left\{min\{\overline{\theta}(\mu)(y),1\}\right\}$$
$$=sup_{x=y\alpha z}\{\overline{\theta}(\mu)(z)\}$$
$$\leq sup_{x=y\alpha z}\{\overline{\theta}(\mu)(y\alpha z)\}$$
$$=sup_{x=y\alpha z}\{\vee_{a\in[y\alpha z]_\theta}\mu(a)\}$$
$$=\vee_{a\in[y\alpha z]_\theta}\mu(a)$$
$$=\vee_{a\in[x]_\theta}\mu(a)$$
$$=\overline{\theta}(\mu)(x)$$

This implies that $\overline{\theta}(\mu)(x)\circ\chi_M\leq\overline{\theta}(\mu)(x)$. Now

$$\overline{\theta}(\mu)\circ\chi_M\circ\overline{\theta}(\mu)(x)=sup_{x=a\alpha b\beta c}\left\{min\{\overline{\theta}(\mu)\circ\chi_M(a\alpha b),\overline{\theta}(\mu)(c)\}\right\}$$
$$\leq sup_{x=a\alpha b\beta c}\left\{min\{\overline{\theta}(\mu)(a\alpha b),\overline{\theta}(\mu)(c)\}\right\}$$
$$=\overline{\theta}(\mu)(x)$$

and $\overline{\theta}(\mu)(x)\circ\chi_M\cap\overline{\theta}(\mu)\circ\chi_M\circ\overline{\theta}(\mu)(x)\leq\left(\overline{\theta}(\mu)\cap\overline{\theta}(\mu)\right)(x)=\overline{\theta}(\mu)(x)$

We obtain $\overline{\theta}(\mu)$ is fuzzy right bi-quasi-ideal *of* M.
Similarly we can prove $\chi_M\circ\underline{\theta}(\mu)\cap\underline{\theta}(\mu)\circ\chi_M\circ\underline{\theta}(\mu)\subseteq\underline{\theta}(\mu)$.
Therefore μ is a θ-upper rough fuzzy bi-quasi-ideal of M.
(ii) Proof is similar to (i).

Theorem 3.2:

Let θ be a congruence relation on M. If μ is a fuzzy bi-quasi-ideal of M . Then,

- μ is a θ-upper rough fuzzy bi-quasi-ideal of M.
- If θ is complete and $\underline{\theta}$ (μ) ≠ ∅ then μ is a θ-lower rough fuzzy bi-quasi-ideal of M.

Proof:

Let μ *is a fuzzy bi-quasi-ideal of* M. Then for all $x\in M,\alpha\in\Gamma$, we have

$$\chi_M\circ\overline{\theta}(\mu)(x)=sup_{x=y\alpha z}\left\{min\{\chi_M(y),\overline{\theta}(\mu)(z)\}\right\}$$
$$=sup_{x=y\alpha z}\left\{min\{1,\overline{\theta}(\mu)(z)\}\right\}$$
$$\leq sup_{x=y\alpha z}\{\overline{\theta}(\mu)(y\alpha z)\}$$
$$=sup_{x=y\alpha z}\{\vee_{a\in[y\alpha z]_\theta}\mu(a)\}$$
$$=\vee_{a\in[x]_\theta}\mu(a)$$
$$=\overline{\theta}(\mu)(x)$$

Then $\chi_M\circ\overline{\theta}(\mu)(x)\leq\overline{\theta}(\mu)(x)$.--(1)

$$\overline{\theta}(\mu)(x)\circ\chi_M\circ\overline{\theta}(\mu)(x)\leq\overline{\theta}(\mu)(x)\circ\overline{\theta}(\mu)(x)=\overline{\theta}(\mu)(x),.\text{--------------------------------(2)}$$

From (1) and (2) we have $\chi_M\circ\overline{\theta}(\mu)(x)\cap\overline{\theta}(\mu)(x)\circ\chi_M\circ\overline{\theta}(\mu)(x)\leq\overline{\theta}(\mu)(x)$.
We obtain $\chi_M\circ\overline{\theta}(\mu)(x)\cap\overline{\theta}(\mu)(x)\circ\chi_M\circ\overline{\theta}(\mu)\subseteq\overline{\theta}(\mu)$. This implies that $\overline{\theta}(\mu)$ is a left bi-quasi-ideal of *M*.
Similarly we have $\overline{\theta}(\mu)\circ\chi_M\cap\overline{\theta}(\mu)\circ\chi_M\circ\overline{\theta}(\mu)\subseteq\overline{\theta}(\mu)$, then that $\overline{\theta}(\mu)$ is a right bi-quasi-ideal of *M*.
Therefore $\overline{\theta}(\mu)$ is a bi-quasi-ideal of *M*.

(ii) Similar to (i)

Theorem 3.3:

Let θ be a congruence relation on M. If μ and λ are fuzzy bi-quasi-ideals of M, then θ(μ) ∩ θ(λ) is a rough fuzzy bi-quasi-ideal of M.

Proof:

Let μ and λ be fuzzy left bi-quasi-ideal of M. Then $\overline{\theta}(\mu)$ and $\overline{\theta}(\lambda)$ are bi-quasi- ideal of M and $x \in M, \alpha \in \Gamma$ we have

$$\chi_M \circ \left(\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)\right)(x)$$
$$= sup_{\ x=y\alpha z} \left\{min\ \{\chi_M(y), (\overline{\theta}(\mu) \cap \overline{\theta}(\lambda))(z)\}\right\}$$
$$= sup_{\ x=y\alpha z} \left\{min\ \{\chi_M(y), min\ \{\vee_{p\in[z]_\theta}\mu(p), \vee_{p\in[z]_\theta}\lambda(p)\}\}\right\}$$
$$= sup_{\ x=y\alpha z} \left\{min\ \{min\ \{\chi_M(y), \vee_{p\in[z]_\theta}\mu(p)\}, min\ \{\chi_M(y), \vee_{p\in[z]_\theta}\lambda(p)\}\}\right\}$$
$$= min\ \left\{sup_{\ x=y\alpha z} \{min\ \{\chi_M(y), \vee_{p\in[z]_\theta}\mu(p)\}\}, su\ p_{x=y\alpha z}\ \{min\ \{\chi_M(y), \vee_{p\in[z]_\theta}\lambda(p)\}\}\right\}$$
$$= min\ \left\{sup_{\ x=y\alpha z} \{min\ \{\chi_M(y), \overline{\theta}(\mu)(z)\}\}, sup_{\ x=y\alpha z} \{min\ \{\chi_M(y), \overline{\theta}(\mu)(z)\}\}\right\}$$
$$= min\ \{\chi_M \circ \overline{\theta}(\mu)(x), \chi_M \circ \overline{\theta}(\lambda)(x)\}$$
$$= \chi_M \circ \overline{\theta}(\mu)(x) \cap \chi_M \circ \overline{\theta}(\lambda)(x)$$

Therefore $\chi_M \circ \left(\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)\right)(x) = \chi_M \circ \overline{\theta}(\mu)(x) \cap \chi_M \circ \overline{\theta}(\lambda)(x)$. Now

$$\left(\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)\right) \circ \chi_M \circ \left(\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)\right)(x)$$
$$= sup_{\ x=a\alpha b\beta c} \left\{min\ \{\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)(a), \chi_M \circ \overline{\theta}(\mu) \cap \overline{\theta}(\lambda)(b\beta c)\}\right\}$$
$$= sup_{\ x=a\alpha b\beta c} \left\{min\ \{min\ \{\overline{\theta}(\mu)(a), \overline{\theta}(\lambda)(a)\}, \chi_M \circ min\ \{\overline{\theta}(\mu)(b\beta c), \overline{\theta}(\lambda)(b\beta c)\}\}\right\}$$
$$= sup_{\ x=a\alpha b\beta c} \left\{min\ \{min\ \{\overline{\theta}(\mu)(a), \overline{\theta}(\lambda)(a)\}, min\ \{, \chi_M \circ \overline{\theta}(\lambda)(b\beta c)\}\}\right\}$$
$$= sup_{\ x=a\alpha b\beta c} \left\{min\ \{min\ \{\overline{\theta}(\mu)(a), \chi_M \circ \overline{\theta}(\mu)(b\beta c)\}, min\ \{\overline{\theta}(\lambda)(a), \chi_M \circ \overline{\theta}(\lambda)(b\beta c)\}\}\right\}$$
$$= sup_{x=a\alpha b\beta c} \{min\{min\{\vee_{p\in[a]_\theta}\mu(p), \chi_M \circ \vee_{p\in[b\beta c]_\theta}\mu(b\beta c)\}$$
$$, min\{\vee_{p\in[a]_\theta}\lambda(p), \chi_M \circ \vee_{p\in[b\beta c]_\theta}\lambda(b\beta c)\}\}\}$$
$$= min\left\{sup_{x=a\alpha b\beta c} \{min\{\vee_{p\in[a]_\theta}\mu(p), \chi_M \circ \vee_{p\in[b\beta c]_\theta}\mu(b\beta c)\}\},\right.$$
$$\left. sup_{x=a\alpha b\beta c} \{min\{\vee_{p\in[a]_\theta}\lambda(p), \chi_M \circ \vee_{p\in[b\beta c]_\theta}\lambda(b\beta c)\}\}\right\}$$
$$= min\ \{sup_{\ x=a\alpha b\beta c}\ \{min\{\overline{\theta}(\mu)(a), \chi_M \circ \overline{\theta}(\mu)(b\beta c)\}\},$$
$$sup_{x=a\alpha b\beta c} \{min\{\overline{\theta}(\lambda)(p), \chi_M \circ \overline{\theta}(\lambda)(b\beta c)\}\}\} =$$
$$min\ \{\overline{\theta}(\mu) \circ \chi_M \circ \overline{\theta}(\mu), \overline{\theta}(\lambda) \circ \chi_M \circ \overline{\theta}(\lambda)\}$$
$$= \overline{\theta}(\mu) \circ \chi_M \circ \overline{\theta}(\mu), \overline{\theta}(\lambda) \circ \chi_M \circ \overline{\theta}(\lambda).$$

Therefore $\left(\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)\right) \circ \chi_M \circ \left(\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)\right)(x) = \overline{\theta}(\mu) \circ \chi_M \circ \overline{\theta}(\mu) \cap \overline{\theta}(\lambda) \circ \chi_M \circ \overline{\theta}(\lambda)$.

Hence

$$\chi_M \circ \overline{\theta}(\mu) \cap \overline{\theta}(\lambda) \cap \overline{\theta}(\mu) \cap \overline{\theta}(\lambda) \circ \chi_M \circ \overline{\theta}(\mu) \cap \overline{\theta}(\lambda) = \chi_M \circ \overline{\theta}(\mu) \cap \overline{\theta}(\mu) \circ \chi_M \circ \overline{\theta}(\mu) \cap$$
$$\chi_M \circ \overline{\theta}(\lambda) \cap \overline{\theta}(\lambda) \circ \chi_M \circ \overline{\theta}(\lambda) \subseteq \overline{\theta}(\mu) \cap \overline{\theta}(\lambda)$$

Hence $\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)$ is a fuzzy left bi-quasi-ideal of M.

Similarly we can prove $\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)$ is a fuzzy right bi-quasi-ideal of M.

Thus $\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)$ is a fuzzy bi-quasi-ideal of M.

Similarly we can prove $\underline{\theta}(\mu) \cap \underline{\theta}(\lambda)$is a fuzzy bi-quasi-ideal of M.

Therefore $\theta(\mu) \cap \theta(\lambda)$ is a rough fuzzy bi-quasi-ideal *of* M.

Theorem 3.4:

Let θ be a congruence relation on M. If μ and λ be fuzzy right ideal and fuzzy left ideal of M respectively, then θ(μ) ∩ θ(λ) is a rough fuzzy bi-quasi-ideal of M.

Proof:

Let μ and λ be fuzzy right ideal and fuzzy left ideal of M respectively. Then $\overline{\theta}(\mu)$ be a fuzzy right ideal and $\overline{\theta}(\lambda)$ be fuzzy right ideal of M. By Theorem 3.3 we have

$\chi_M \circ \left(\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)\right) = \chi_M \circ \overline{\theta}(\mu) \cap \chi_M \circ \overline{\theta}(\lambda)$ and

$\left(\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)\right) \circ \chi_M \circ \left(\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)\right) = \overline{\theta}(\mu) \circ \chi_M \circ \overline{\theta}(\mu) \cap \overline{\theta}(\lambda) \circ \chi_M \circ \overline{\theta}(\lambda)$.

$\Rightarrow \chi_M \circ \left(\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)\right) \cap \left(\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)\right) \circ \chi_M \circ \left(\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)\right)$.

$$= \left(\chi_M \circ \overline{\theta}(\mu)\right) \cap \left(\overline{\theta}(\mu) \circ \chi_M \circ \overline{\theta}(\mu)\right) \cap \left(\chi_M \circ \overline{\theta}(\lambda)\right) \cap \left(\overline{\theta}(\lambda) \circ \chi_M \circ \overline{\theta}(\lambda)\right)$$
$$\subseteq \overline{\theta}(\mu) \cap \overline{\theta}(\lambda)$$

Hence $\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)$ is a fuzzy left bi-quasi-ideal of M. Similarly we have $\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)$ is a fuzzy right bi quasi -ideal of M. Therefore $\overline{\theta}(\mu) \cap \overline{\theta}(\lambda)$ is a fuzzy bi-quasi-ideal of M.
In the same way we have proved $\underline{\theta}(\mu) \cap \underline{\theta}(\lambda)$ is a fuzzy bi-quasi-ideal of M.
Hence $\theta(\mu) \cap \theta(\lambda)$ is a rough fuzzy bi-quasi-ideal of M.

Theorem 3.5:

Let θ be a congruence relation on M. If μ is a fuzzy quasi-ideal of M . Then θ(μ) is a rough ideal of M.

Proof:

Let μ be a fuzzy quasi-ideal of M. Then $\overline{\theta}(\mu)$ be a fuzzy quasi-ideal of M. For all $x \in M, \alpha \in \Gamma$, we have

$\chi_M \circ \overline{\theta}(\mu)(x) = \chi_M \circ \overline{\theta}(\mu) \cap \overline{\theta}(\mu) \circ \chi_M(x) \leq \overline{\theta}(\mu)(x)$, since $\overline{\theta}(\mu)$ is a fuzzy quasi-ideal of M.
Thus $\chi_M \circ \overline{\theta}(\mu) \leq \overline{\theta}(\mu)(x)$. Similarly we have $\overline{\theta}(\mu) \circ \chi_M(x) \leq \overline{\theta}(\mu)(x)$
Therefore $\overline{\theta}(\mu)$ is a fuzzy ideal of M.
In the same way we have proved $\underline{\theta}(\mu)$ is a fuzzy ideal of M.
Hence $\theta(\mu)$ is a rough ideal of M.

Theorem 3.6:

Let θ be a congruence relation on M. Then M is regular if and only if $\theta(\lambda) \circ \theta(\mu) = \theta(\lambda) \cap \theta(\mu)$ for any fuzzy right ideal λ and fuzzy left ideal μ of M .

Proof:

Let λ be fuzzy right ideal and μ be fuzzy left ideal of M. Then $\overline{\theta}(\lambda)$ be fuzzy right ideal and $\overline{\theta}(\mu)$ be fuzzy left ideal of M. For all $x \in M, \alpha \in \Gamma$, we have

$$\overline{\theta}(\lambda) \circ \overline{\theta}(\mu)(x) = sup_{\ x=y\alpha z} \left\{min \ \{\overline{\theta}(\lambda)(y), \overline{\theta}(\mu)(z)\}\right\}$$
$$\leq min \ \left\{sup_{\ x=y\alpha z} \ \{\overline{\theta}(\lambda)(y\alpha z), \overline{\theta}(\mu)(y\alpha z)\}\right\}$$
$$= min \ \left\{sup_{\ x=y\alpha z} \ \{\vee_{p\in[y\alpha z]_\theta} \lambda(p), \vee_{p\in[y\alpha z]_\theta} \mu(p)\}\right\}$$
$$= min \ \{\vee_{p\in[x]_\theta} \lambda(p), \vee_{p\in[x]_\theta} \mu(p)\}$$
$$= min \ \{\overline{\theta}(\lambda)(x), \overline{\theta}(\mu)(x)\}$$
$$= \overline{\theta}(\lambda) \cap \overline{\theta}(\mu)(x)$$

Then $\overline{\theta}(\lambda) \circ \overline{\theta}(\mu) \subseteq \overline{\theta}(\lambda) \cap \overline{\theta}(\mu)$.

Consider $\overline{\theta}(\lambda) \cap \overline{\theta}(\mu)(x) = min \ \{\overline{\theta}(\lambda)(x), \overline{\theta}(\mu)(x)\}$

$$\leq min \ \left\{sup_{\ x=y\alpha z} \ \{\overline{\theta}(\lambda)(y), \overline{\theta}(\mu)(z)\}\right\}$$
$$\leq sup_{\ x=y\alpha z} \ \left\{min \ \{\overline{\theta}(\lambda)(y), \overline{\theta}(\mu)(z)\}\right\}$$
$$= \overline{\theta}(\lambda) \circ \overline{\theta}(\mu)$$

Then $\overline{\theta}(\lambda) \cap \overline{\theta}(\mu) \subseteq \overline{\theta}(\lambda) \circ \overline{\theta}(\mu)$.
Similarly we have proved $\underline{\theta}(\lambda) \cap \underline{\theta}(\mu) \subseteq \underline{\theta}(\lambda) \circ \underline{\theta}(\mu)$.
Therefore $\theta(\lambda) \circ \theta(\mu) = \theta(\lambda) \cap \theta(\mu)$.

Theorem 3.7:

Let θ be a congruence relation on M. Then M is regular if and only if $\theta(\mu) = \chi_M \circ \theta(\mu) \cap \theta(\mu) \circ \chi_M \circ \theta(\mu)$ for any fuzzy bi-quasi- ideal μ of M.

Proof:

Let μ be a fuzzy bi-quasi-ideal of regular Γ-semigroup M. By Theorem 3.3 $\overline{\theta}(\mu)$ fuzzy bi-quasi-ideal of M.Then $\chi_M \circ \overline{\theta}(\mu) \cap \overline{\theta}(\mu) \circ \chi_M \circ \overline{\theta}(\mu) \subseteq \overline{\theta}(\mu)$.-----------------------------(1)
For all $x \in M, \alpha \in \Gamma$,

$$\chi_M \circ \overline{\theta}(\mu)(x) = sup_{\ x=x\alpha y\beta x} \ \left\{min \ \{\chi_M(x\alpha y), \overline{\theta}(\mu)(x)\}\right\} = \overline{\theta}(\mu)(x).$$
$$\overline{\theta}(\mu) \circ \chi_M \circ \overline{\theta}(\mu)(x) = sup_{\ x=x\alpha y\beta x} \ \left\{min \ \{\overline{\theta}(\mu)(x), \chi_M \circ \overline{\theta}(\mu)(y\beta x),\}\right\}$$

$$= sup_{x = x\alpha y\beta x} \left\{ min \left\{ \overline{\theta}(\mu)(x), sup_{y\beta x = \alpha y b} \left\{ min \left\{ \chi_M(a), \overline{\theta}(\mu)(b) \right\} \right\} \right\} \right\}$$

$$= sup_{x = x\alpha y\beta x} \left\{ min \left\{ \overline{\theta}(\mu)(x), sup_{y\beta x = \alpha y b} \left\{ min \left\{ 1, \overline{\theta}(\mu)(b) \right\} \right\} \right\} \right\}$$

$$\geq sup_{x = x\alpha y\beta x} \left\{ min \left\{ \overline{\theta}(\mu)(x), \overline{\theta}(\mu)(x) \right\} \right\} = \overline{\theta}(\mu)(x) .$$

Then $\overline{\theta}(\mu) \subseteq \chi_M \circ \overline{\theta}(\mu) \cap \overline{\theta}(\mu) \circ \chi_M \circ \overline{\theta}(\mu)$------------------------------(2)

From (1) and (2) we have $\chi_M \circ \overline{\theta}(\mu) \cap \overline{\theta}(\mu) \circ \chi_M \circ \overline{\theta}(\mu) = \overline{\theta}(\mu)$.

In the similar way we have to prove $\chi_M \circ \underline{\theta}(\mu) \cap \underline{\theta}(\mu) \circ \chi_M \circ \underline{\theta}(\mu) = \underline{\theta}(\mu)$.

Therefore $\theta(\mu) = \chi_M \circ \theta(\mu) \cap \theta(\mu) \circ \chi_M \circ \theta(\mu)$.

Conversely suppose that $\chi_M \circ \overline{\theta}(\mu) \cap \overline{\theta}(\mu) \circ \chi_M \circ \overline{\theta}(\mu) = \overline{\theta}(\mu)$ for any fuzzy bi-quasi-ideal $\overline{\theta}(\mu)$ of M.

Let B be the bi-quasi-ideal of M. Then $\overline{\theta}(B)$ is a bi-quasi-ideal of M. By Theorem [2.4] $\chi_{\overline{\theta}(B)}$ is a fuzzy bi-quasi-ideal of M. We have $\chi_{\overline{\theta}(B)} = \chi_M \circ \chi_{\overline{\theta}(B)} \cap \chi_{\overline{\theta}(B)} \circ \chi_M \circ \chi_{\overline{\theta}(B)} = \chi_{M\Gamma\overline{\theta}(B)} \cap \chi_{\overline{\theta}(B)\Gamma M\Gamma\overline{\theta}(B)}$ and $\overline{\theta}(B) = M\Gamma\overline{\theta}(B) \cap \overline{\theta}(B)\Gamma M\Gamma\overline{\theta}(B)$. By Theorem[2.3] in [8] M is regular. This argument is true for $\underline{\theta}(\mu)$.

5. Conclusion:

The rough set theory is regarded as a generalization of the classical set theory. A key notion in rough set is an equivalence relation. An equivalence is sometime difficult to be obtained in reward problems due to vagueness and incompleteness of human knowledge. In this paper we introduce the concept of rough bi-quasi ideals and characterize rough bi-quasi simple Γ -semigroup and regular Γ -semigroup. We plan to study rough fuzzy bi-quasi-ideals in rings.

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