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A STUDY ON GSM – MOBILE PHONE NETWORK IN GRAPH THEORY

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Introduction:

"Graph theory" is an important branch of mathematics, (Euler 1707-1782) is known as the father of Graph theory as well as Topology. Graph theory came into existence during the first half of the 18^{th} century. Graph theory did not start to develop into an organized branch of mathematics until the second half of the 19^{th} century and there was not even a book on the subject until the first half of the 20^{th} century. Graph theory has experienced a tremendous growth, one of the main reason for this phenomena is the as Physics, Chemistry, Biology, Psychology, Sociology and theoretical Computer science. It has grown rapidly in recent times with a lot of research activities. Today, at the international level, one third of the mathematics research papers are from Graph theory and Combinatories. The Significance and importance of graph theory is its immense applications of other fields. The blossoming of a new branch study in the field of chemistry. "Chemical Graph theory" is get another proof of the importance and role of Graph theory. In Physics, Graph theory is applied in continuum Statistical Mechanics and desecrates statistical mechanics Graph models have been used to study Polymer chains of hydro0carbons and percolation theory. It is shown that the calculation of the energy of spin configuration leads to the Chinese problem. Electrical Engineering is an area where graph had found its earliest application. In 1847, Kirchhoff introduced graph models considering the edges as passive electrical elements (resistors, capacitors and inductors) and the nodes as the junctions where two or more element met. The role of Graph theory in Computer science everywhere. In Computer science, Graph theoretic models are applicable to Computer languages, circuits and switching theory, Computer networks and reliability, inter connection networks for parallel processors such as diagnostic graphs, very large scale integrated designs can be related with planer graph models. Graph models. Graph models are very useful in distributed computer system and parallel computing. Here, Petri nets play a vital role. These can be used to characterize minimal deadlocks. Growth of Graph theory is mainly due to the advent of computers. Graph theory plays an important role in several areas of Computer Science.

GSM – Mobile Phone Network:

Definition: A graph is a diagram consisting of points called vertices, joined by directed lines called arcs, each arc joining exactly two vertices.

Example:

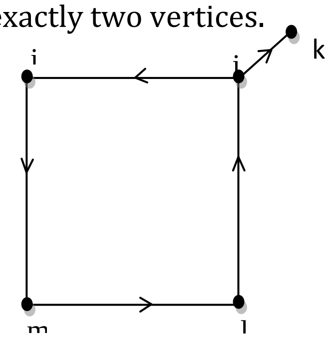

Figure 1

Definition: The in degree $d^-(v)$ of a vertex v in a digraph D is the number of arcs of a vertex v in a digraph D is the number of arcs having v as its terminal vertex. The out degree $d^+(v)$ of v is the number of arc having v as its initial vertex. The ordered pair $(d^+(v), d^-(v))$ is called the degree pair of v.

Definition: A Undirected graph G = (V, E) consists of a finite set of vertices v and a set of edges E. If differs from a directed graph in that each edge in E is an unordered pair of vertices. If (V, W) = (W, V).

Example:

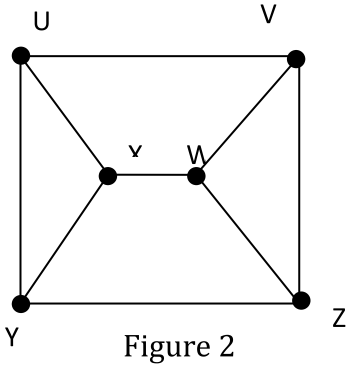

Figure 2

Definition: A graph is a weighted graph, if there is a real number associated with each edge of G.

Example:

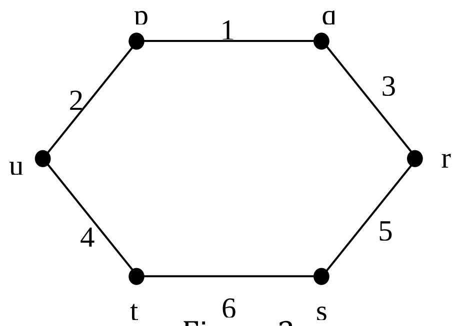

Figure 3

Definition: GSM, which stands for Global system for mobile communications, regions as the world's most widely used cell phone technology. Cell phones use a cell phone service carrier's GSM network by searching for cell phone towers in the nearly.

Definition: The connectivity k = k(G) of a graph G is the minimum number of points whose removal result in a disconnected or trivial graph. The line connectivity $\lambda = \lambda(G)$ of g is the minimum number of lines whose removal results in a disconnected or trivial graph.

Definition: An assignment of colors to the vertices of a graph so that no two adjacent vertices get the same color is called a coloring of the graph. For each color, the set of all points which get that color is independent and is called a color class. A coloring of graph G using at most n colors is called an n coloring. The chromic number $\chi(G)$ of graph G is the minimum number of colors needed to color G. A graph G is called, n-colourable if $\chi(G) \leq n$.

Four Colour Problems:

The four colour conjecture states that map on the surface of a sphere can be coloured with only four colors so that no two adjacent countries have the same colour. Each country must consist of single connected region adjacent countries are those having a boundary line (not merely a single point) in common. The problem of deciding whether the four conjectures is true or false is called four colour problem. A plane graph (geometric dual) can be associated with each map. Colouring the countries of the map is equivalent to coloring the vertices of its Geometric dual. In this set up, the four colour conjecture states that "Every planar graph is 4-colourable".

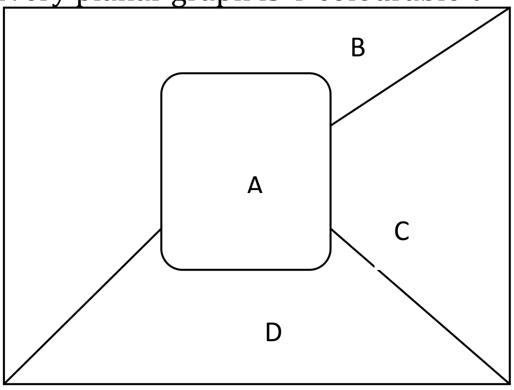

Figure 4

The number 4 cannot be reduced further as there are maps that require at least four colours. The map in Fig: 2.4 is one such map. There are several problems in graph theory that are equivalent to the colour problem. One of these is the case n = 5 of the following conjecture.

Theorem:

In a digraph d, sum of the in degrees of all the vertices is equal to the sum of their out degree, each sum being equal to the number of arcs in D.

Proof:

Let q denote the number of arcs D = (V, A). Let B = $\sum d^{-}(v)$ and C = $\sum d^{+}(v)$. An arc (u, w) contributes one to the out degree of w. Hence each arc contributes 1 to the sum B and 1 to the sum c. Hence B = C = q.

General Communication Networks:

So for we have considered communication networks in which the weight associated with a directed edge represents the probability of communication along that edge. We can however have more general networks.

- ✓ For communication of messages where the directed edge represents the channel and the weight represents the capacity of the channel say in bits per second.
- ✓ For communication of gas in pipeline where the weights are the capacities, say in gallons per hour.
- ✓ Communication roads where the weights are the capacities, say in gallons per hour.

An interesting problem is to find the maximum flow rate, of whatever is being communicated, from any vertex of the communication network to any other. Useful graph-theoretic algorithms for this have developed by Elias. Feinstein and Shannon as well as by Ford and Fulkerson.

Communication Networks:

A directed graph can serve as a model for a communication network. Thus consider the network.

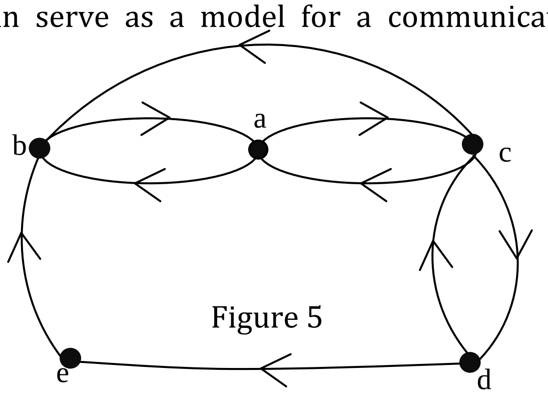

Figure 5

If an edge is directed from a to b, it means that a can communicate with b. In the given network e can communicate directly with b, but b can communicate with e only indirectly through c and d .However every individual can communicate with every other individual. Our problem is to determine the importance of each individual in this network. The importance can be measured by the fraction of the messages on an average that pass through him. In the absence of any other knowledge, we can assume that if individual can send message direct to n individual, he will send a message to any one of them with probability 1/n. In the present example, the communication probability matrix is

a	b	c	d	e	
a	0	1/2	1/2	0	0
b	1/2	0	1/2	0	0
c	1/3	1/3	0	1/3	0
d	0	0	1/2	0	1/2
e	0	1	0	0	0

No individual is to send a message to himself and so all diagonal elements are zero. Since all elements of the matrix are non-negative and the sum of elements of every row is unity. The matrix is a stochastic matrix and one of its Eigen values is unity. In the long run, these fractions of message will pass through a, b, c, d, e respectively. Thus we can conclude that in this network, C is the most important person. If in a network, an individual cannot communicate with every other individual either directly or indirectly, the Markov chain is not ergodic and the process of finding the importance of each individual breaks down.

Communication Networks with Known Probabilities of Communication:

In the communication graph of Figure 2.5, we know that a can communicate with both b and c only and in the absence of any other knowledge, we assigned equal probabilities to a's chances of communicating with b and c are in the ratio 3:2 then we assign probability 0.6 to a's communicating with b and 0.4 to a's communicating with c. Similarly we can associate a probability with every directed edge and we get the weighted digraph (figure 2.6) with the associated matrix.

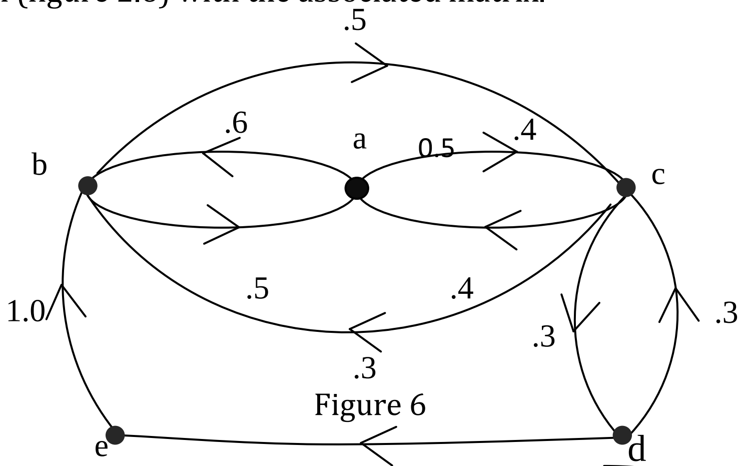

Figure 6

a	b	c	d	e	
a	0	0.6	0.4	0	0
b	0.5	0	0.5	0	0
B =	c	0.4	0.3	0	0.3
d	0	0	0.3	0	0.7
e	0	1.0	0	0	0

We note that the elements are all non-negative and the sum of the elements of every row is unity so that B is a stochastic matrix and unity is one of its eigen values. The eigenvector corresponding to this eigen values will be different from the Eigen vector found communication network and so the relative importance of individual depends both on the directed edges as well as one the weights associated with the edges.

Application of GSM Mobile Phone Networks:

The Group special Mobile (GSM) was created in 1982 to provide a standard for a mobile telephone system. The first GSM network was launched in 1991 by Radiolinja in Finland. Today, GSM is the most popular standard for mobile phones in the world, used by over 2 billion people across more than 212 countries. GSM is cellular network with its entire geographical range divided into hexagonal cells. Each cell has a communication tower which connects with mobile phones within the cell. All mobile phone connect to the GSM network by searching for cells in the immediate vicinity. GSM networks operate in only four different frequency ranges. The reason why only four different frequencies suffice is clear. The map of the cellular regions can be properly colored by using only four different colours. That is the map of India is coloured with a minimum of four colours only. Here regions sharing the same colour to share the same frequency. so, the vertex colouring may be used for assigning at most four different frequencies for any GSM mobile phone network.

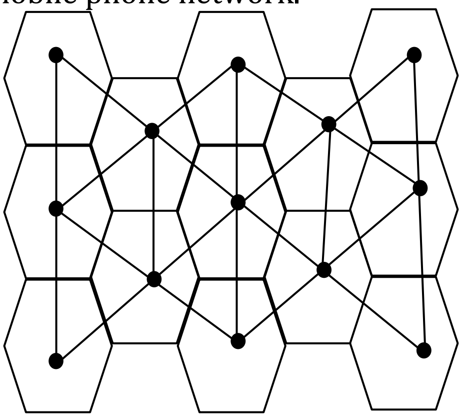

Figure 7

The cells of a GSM mobile phone network

Service on the GSM Network:

There are a number of services available via GSM such as

- ✓ Telephone
- ✓ CSD (Circuit Switched Data, data transfer
- ✓ SMS (Short Message Service)
- ✓ MMS (Multimedia Message Service)
- ✓ FAX
- ✓ GPRS (General Packet Radio service)

Benefits of GSM:

The Global system for mobile communications or GSM is the standard by which the majority of mobile phones operate across the global. As of 2010, there are over 3 billion people in 212 countries operating on the GSM standard understanding the performance benefits of GSM helps to understand why over 80 percent of mobile phone across the world operates on the GSM. For practical and everyday purposes, GSM offer users wider international roaming capabilities then other U.S network technologies and can enable a cell phone to be a “world phone”. More advanced GSM incorporates the earlier TDMA stand. GSM carriers have roaming contracts with other GSM carriers and typically contracts with other GSM carriers and typical and typically cover rural areas more completely then CDMA carries. GSM also advantage of using sim (subscriber identity module) cards in the U.S. The sim card, which acts as your digital identity is tried to your cell phone service carrier's network rather than to the hand set itself. This allows for easy exchange from one phone to another without new cell phone service activation GSM uses digital technology and is a second geration (2G) cell phone system.

Theorem:

If G is not connected then $\bar{G}$ is connected.

Proof:

Since G is not connected, G has more than one component. Let u,v be any two points of G. we will prove that there is a u-v path in $\bar{G}$. Let u ,v belong to different component in G, they are not adjacent in G and hence they are adjacent in $\bar{G}$. If u, v lies in the same component of G, choose W in a different component then u, w, v is a u-v path in $\bar{G}$. Hence $\bar{G}$ is connected.

Types Of Networks::

Social and Economic Networks:

A set of people or groups of people with some pattern of contacts or interactions between them.

Information Networks:

Connections of "information" objects.

Technological Networks:

Designed typically for distribution of a commodity or service.

- ✓ Infrastructure networks: e.g: Internet, Transport network power grid.
- ✓ Temporary networks: e.g: sensor networks, autonomous vehicles.

Genetical Networks:

Genes are classified into f_1, f_2, f_3.... Generations and also it can be classified into father and mother side.

Biological Networks:

A number of biological systems can also be represented as networks. Eg: Food web, protein interaction network.

GRAPH – 1

We represent a network by a graph (N,g) which consists of a set of nodes N={1,2...n} and an n x n matrix $g=[g_{ij}]$ i,j∈N (referred to as an adjacency matrix) where $g_{ij} \in \{0,1\}$ represents the availability of an edge from node i to node j. The edge weight $g_{ij}>0$ can also take on non-binary values, representing the intensity of the interaction, in which case we refer to (N,g) as a weighted graph. We refer to a graph as a directed graph (or digraph) if $g_{ij} \neq g_{ji}$ and an undirected graph if $g_{ij} = g_{ji}$ for all i,j∈N

Example:

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \Rightarrow$$

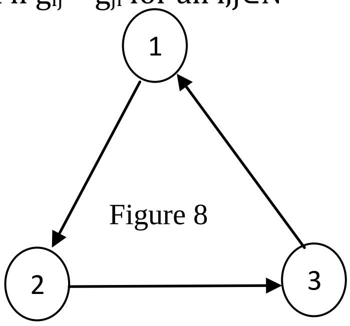

Figure 8

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

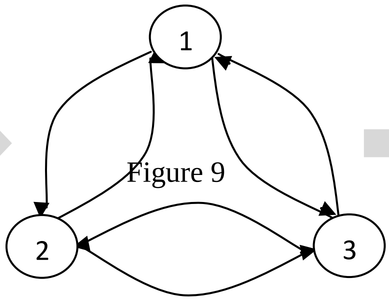

Figure 9

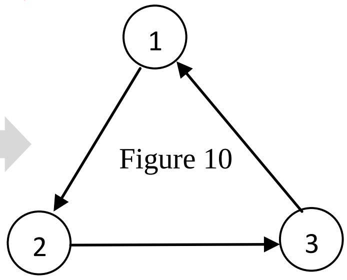

Figure 10

Graph – 2

Another representation of a graph is given by (N,E) where E is the set of edges in the network.

For Directed Graphs: E is the set of "directed" edges i.e., $(i,j) \in E$

For Undirected Graphs: E is the set of "undirected" edges ie., $\{i,j\} \in E$.

In example 3.2.1, $E_d = \{(1,2), (2,3), (3,1)\}$

In example 3.2.2, $E_u = \{\{1,2\},\{1,3\},\{2,3\}\}$

We will use the terms networks and graph interchangeably.

We will sometimes use the notation $(i,j) \in g$ or $\{i,j\} \in g$ to denote $g_{ij} = 1$

We consider "sequences of edges" to capture indirect interactions.

For an Undirected Graph (Ng):

A walk is a sequence of edges $\{i_1,i_2\}, \{i_2,i_3\},\ldots,(i_{k-1},i_k\}$.

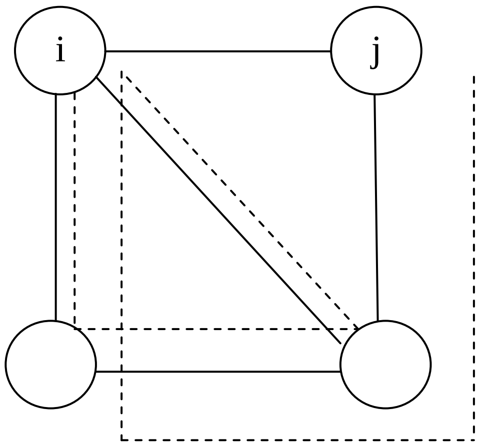

Figure 10: Walk

A path between nodes i and j is a sequence of edges $\{i_1,i_2\}, \{i_2,i_3\},\ldots, \{i_{k-1},i_k\}$ such that $i_1=i$ and $i_k=j$ and each node in the sequence $i_1,i_2,\ldots i_k$ is distinct.

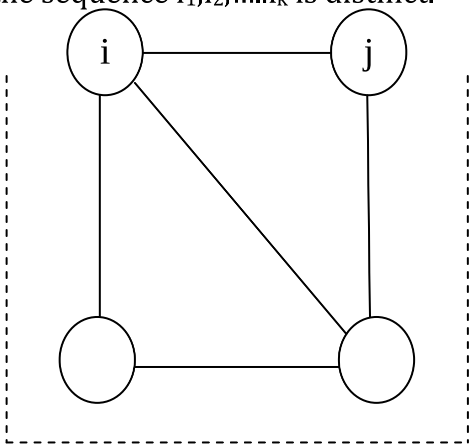

Figure 11: Path between i & j

A cycle is a path with a final edge to the initial edge.

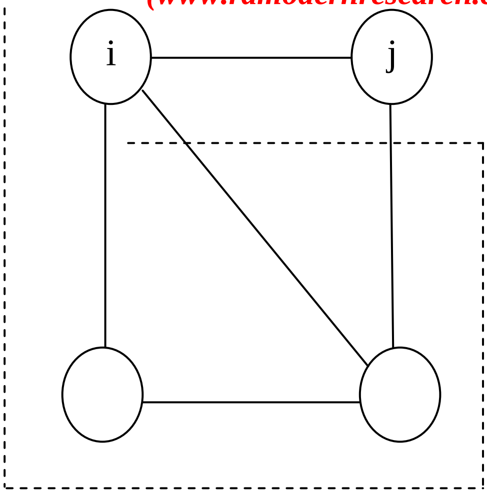

Figure 12: Cycle

A geodesic between nodes i and j is a "shortest path" (i.e. , with minimum number of edges) between these nodes.

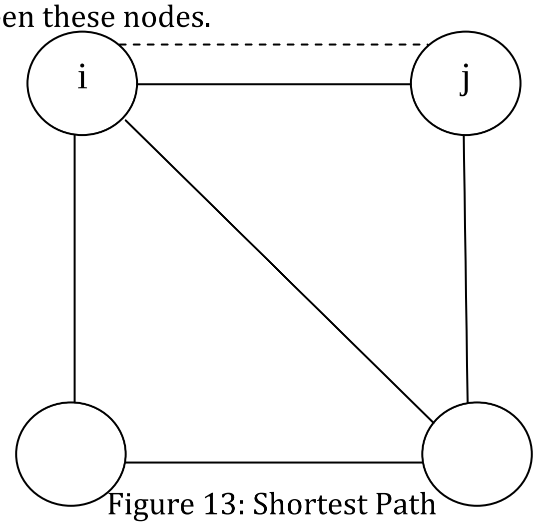

Figure 13: Shortest Path

A path is a walk (or a path) is the number of edges on that walk (or path). For directed graphs, the same definitions hold with directed edges (in which case we say "a path from node i to node j")

Connectivity and Components:

An undirected graph is connected if every two nodes in the network are connected by some path in the network. Components of a graph (or network) are the distinct maximally connected sub graphs. A directed graph is

- ✓ Connected if the underlying undirected graph is connected (i.e., ignoring the directions of edges.)
- ✓ strongly connected if each node can reach every other node by a "directed path"

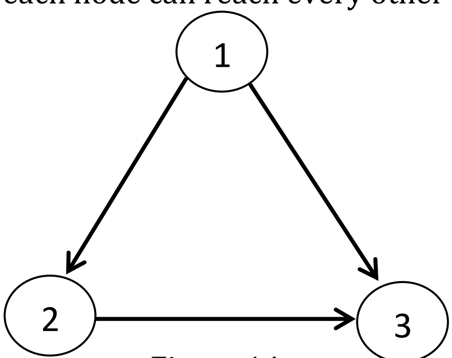

Figure 14

A directed graph that is connected but not strongly connected.

Neighbourhood and Degree of a Node:

The neighborhood of node i is the set of nodes that i is connected to.

For Undirected Graphs: The degree of node is the number of edges that involve I (i.e., Cardinality of his neighbourhood)

For Directed Graphs: Node i's in – degree is $\sum_{J} g_{ji}$

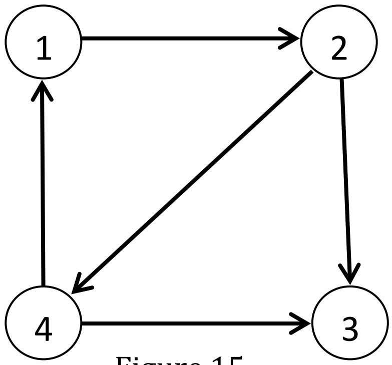

Figure 15

Node 1 has in –degree 1 and out – degree 1

Applications:

Designing One Way Traffic System:

Here the problem is to restrict all the streets of a city to one way traffic in such a way that travel from any point to any other point in the city is possible. Considering the meeting points of two or more streets as edges, we get a diagram. This diagram is either a graph or it has loops or multiple edge. In the later case, it can be transformed to a graph by introducing points of degree 2 on edges wherever necessary. In any case, we get a graph G representing the street system in the city. "Restricting the streets to one way traffic such that travel from any point to any other point is possible" is equivalent to giving directions to the edges of G such that the resulting digraph is strongly connected. The condition under which such an orientation of G is possible is given in the theorem.

Theorem:

For any graph G, $X(G) \leq 1+ \max \delta(G')$ where the maximum is taken over all induced sub graphs G' of G.

Proof:

The theorem is obvious for totally disconnected graphs. Now let G be an arbitrary n-chromatic graph, n≥2. Let H be any smallest induced sub graph of G such that X(H) = n. Hence X(H-v) = n-1 for every point V of H. If $deg_H\ v < n-1$, then a (n-1) colouring of H-v can be extended to a n-1 colouring of H by assigning to v, a colour which is assigned to none its neighbours in H. Hence $deg_H\ v \geq n-1$. Since v is an arbitrary vertex of H, this implies that $\delta(H) \geq n-1 = X(G)-1$. Hence $X(G) \leq 1+\delta(H) \leq 1+\max \delta(H')$ where the maximum is taken over the set A of induced subgraph H' of H.

$$X(G) \leq 1+ \max \delta(G'),$$

Where the maximum is taken over the set B of induced sub graphs G' of G.

One Way Traffic Problems:

The road map of a city can be represented by a directed graph. If only one way traffic is allowed from point a to point b, we draw an edge directed from a to b. If traffic is allowed both ways, we can either draw two edges, one directed from a to b and the other directed from b to a or simply draw an undirected edge between a and b.

The problem is to find whether we can introduce one – way traffic on some or all of the roads without preventing persons from going from any point of the city to any other point. In other words, we have to find when the edges of a graph can be given direction is such a way that there is a directed path from any vertex to every other. It is easily seen that one- way traffic on the road DE cannot be introduced without disconnecting the vertices of the graph.

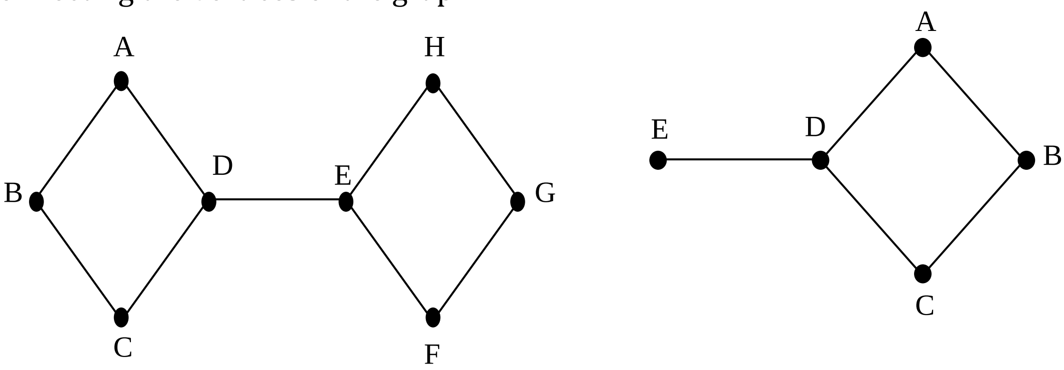

Figure 16

In fig 16, DE can be regarded as a bridge connecting two on which of the town. In fig 16, DE can be regarded as a blind street on which a two – way traffic is necessary. Edges like DE are called separating edges, while other edges are called circuit edges. It is necessary that on separating edges, two –way traffic should be permitted. It can also be shown that this is sufficient. In other words, the following theorem can be established. If G is an undirected connected graph, then one can always direct the circuit edges of G and leave the separating edges undirected (or both way directed)so that there is a directed path from any given vertex to any other vertex.

Electrical Network Problems:

Properties (such as transfer function and input impedance) of an electrical network are functions of only two factors.

- ✓ The nature and value of the elements forming the network such as resistors, inductors, transistors, and so forth.
- ✓ The way these elements are connected together that is the topology of the network.

Since there are only a few different types of electrical elements, the variations in network are chiefly due to variations in topology. Thus electrical network analysis and synthesis are mainly the study of network topology. In the topological study of electrical networks factor 2 is separated from 1 and is studied independently. The advantage of this approach will be clearer in networks.

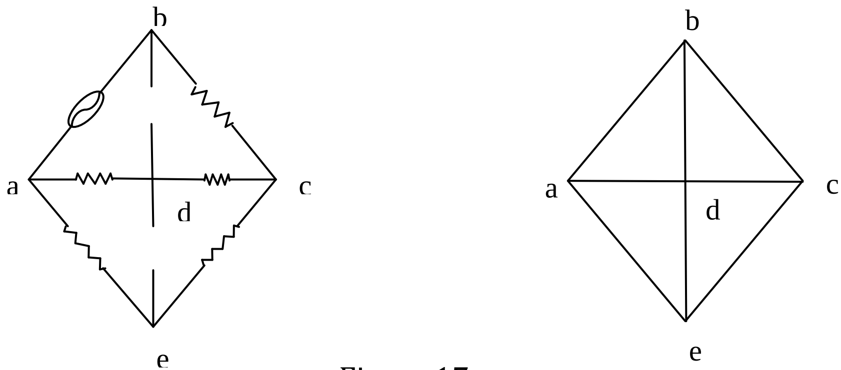

Figure 17

The topology of a network is studies by means of its graph. In drawing a graph of an electrical network the junctions are represented by vertices and branches (which consist are electrical elements) are represented by edges, regardless of the nature and size of the electrical elements. An electrical network and its graph are shown in the figure.

Genetic Graphs:

In a genetic graph, we draw a directed edge from A to B to indicate that B is the child of A. In general each vertex will have two incoming edges, one from the vertex representing the father and the other from the vertex representing the mother. If the father or mother is unknown, there may be less than two incoming edges. Thus in a genetic graph, the local degree of incoming edges at each vertex must be less than or equal or two. This is a necessary condition for a directed graph to be a genetic graph, but it is not a sufficient condition.

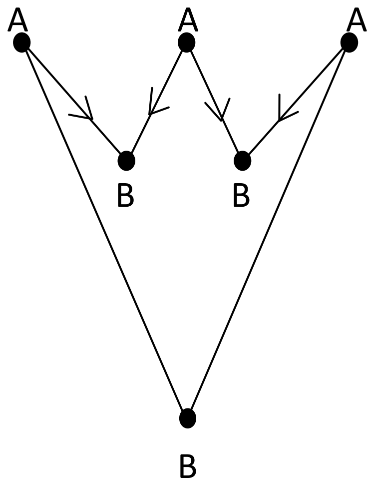

Figure 18

Thus Figure 18 does not give a genetic graph inspite of the fact that the number of incoming edges at each vertex does not exceed two. Suppose A_1 is male, then A_2 must be female. Since A_1, A_2 have a child B_1.Then A_2 must be male, Since A_2, A_3 have a child B_2. Now A_1, A_3 being both males cannot have a child B_3.

Conclusion:

In this paper, we have study about the computer network security in graphs. Dharwadkar's approximation algorithm which efficiently outputs optimal vertex cover sets for many graph classes. In a similar way, we can study another parameters equivalently using these concept, we can define new parameters like GSM-mobile phone networks, information networks and biological networks. Hence, one way traffic system and electrical networks problems results are produced. Likewise, other networks may be find out for using the concept.

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