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A NEW MATHEMATICAL MODEL TO FIND THE ECONOMIC IMPACT OF MALARIA-RELATED DEATH IN ENDEMIC REGIONS

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Abstract:

Malaria Poses one of the most serious and complex problems in the developing countries. It is a heavy burden on human population, a major cause of work loss and a serious impediment to economic development and productivity in endemic regions. It is observed that the economic impact of malaria-related death also varies according to the age of the diseased. Keeping these in mind, an age-dependent malaria model with impact of malaria-related death on economic growth is proposed. This model is analyzed by using qualitative theory of differential equations and numerical simulation. It is concluded that in the highly endemic regions, where most malaria-related deaths are among infants and young children, the impact on economy is lower than in areas of low to moderate endemicity, where the burden of disease falls primarily on adults.

1. Mathematical Modelling:

Mathematical modelling essentially consists of translating real world problems into mathematical problems, solving mathematical problems and interpreting these solutions in the language of the real world.

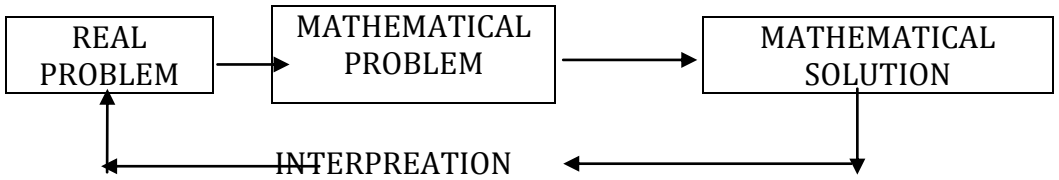


Figure 1: Classification

This is expressed figuratively by saying that we catch hold of the real world problem in our teeth, drive into the mathematical ocean, swim there for some time and we come out to the surface with the solution of the real world problem with us. Alternatively we may say that we soar high into the mathematical atmosphere along with the problem, fly there for some times and come down to the earth with the solution. A real world problem, in all its generality can seldom be translated into a mathematical problem and even if it can be so translated, it may not be possible to solve the resulting mathematical problem. As such it is quite often necessary to 'idealise' or 'simplify' the problem or approximate it by another problem which is quite close to the original problem and yet it can be translated and solved mathematically. In this idealisation, we try to retain all the essential features of the problem, giving up those features which are not very essential or relevant to the situation we are investigating.

Sometimes the idealization assumptions may look quite drastic: Thus for considering the motions of planets, we may consider the planets and sun as point masses and neglect their sizes and structures. Similarly for considering the motion of a fluid, we may treat it as a continuous medium and neglect its discrete nature in terms of its molecular structure. The justification for such assumptions is often be found in terms of the closeness of the agreement between observations and predictions of the mathematical models. This leads is to modify fig- 1 to the following figure 2.

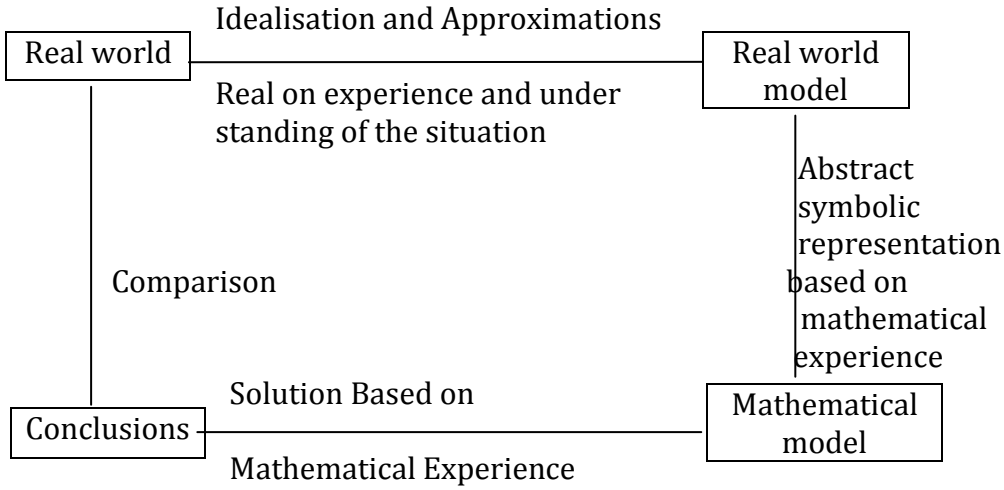


Figure 2

If the comparison is not satisfactory, we modify either the idealisation assumptions or search for another structure for the mathematical model. This leads to the following twelve-point procedure for solving problems through mathematical modeling.

- ✓ Be clear about the real world situation to be investigated. Find all its essential characteristics relevant to the situation and find those aspects which are irrelevant or whose relevance is minimal. It is important to decide what aspects must be considered and what aspect can be ignored.
- ✓ Think about all the physical, chemical, biological, social, economic laws that may be relevant to the situation. If necessary collect some data and analyse it to get some initial insight into this situation.
- ✓ Formulate the problem in problem language.
- ✓ Think about all the variables x_1, x_2, ... x_n and parameters a_1, a_2, ... a_m involved. Classify these into known and unknown ones.
- ✓ Think of the most appropriate mathematical model and translate the problem suitably into mathematical language (ML) in the form $f_i(x_i, ah, \frac{\partial}{\partial x_i}, \int .. dx_i, d) \le 0$. (ie) In terms of algebraic, transcendental, differential, difference, integral, integro-differential, differential difference equations or inequations.
- ✓ Think of all possible ways of solving the equations of the model. The methods may be analytical, numerical or simulation. Try to get as for as possible analytically, supplement this with numerical and computer methods when necessary and use simulation when warranted.
- ✓ If a reasonable change in the assumptions make the analytical solution possible investigate the possibility.
- ✓ Make an error analysis of the method used. if the error is not within acceptable limits, change the method of solution.
- ✓ Translate the final solution into Problem Language.
- ✓ Compare the prediction with available observation or data. If agreement is good, accept the model. If agreement is not good, examine the assumptions and approximations and change them in the light of the discrepancies observed and proceed as before.
- ✓ Continue the process till a satisfactory model is obtained which explains all earlier data and observations.

- ✓ Deduce conclusions from your model and test these conclusions against earlier data additional data and that may be collected and see if the agreement still continues to be good.

2. Mathematical Bioscience and Epidemic Model:

Mathematical Bioscience:

In Mathematical Bioscience, we study the applications of mathematical modelling and mathematical techniques to get an insight into the problems of biosciences. Quite often the term biomathematics is used for Mathematical Bioscience. However, this term is some- times also used in the narrow sense of mathematics for biologist and medical scientists, that is, for a collection of Mathematical techniques specially applicable to bioscience. In this sense, biomathematics has an interpretation similar to engineering mathematics or physical mathematics or social science mathematics, which stand for the mathematical techniques applicable to engineering, physical science, and social science respectively. The terms mathematical biology, theoretical biology or mathematical life sciences are also used. But the term mathematical bioscience is most appropriate and has a wlder scope. It includes mathematical ecology, mathematical demography, mathematical bio economics, mathematical medical science and mathematical agriculture. Some of the disciplines included in mathematical bioscience are:

- ✓ Mathematical botany this is concerned with problem such as cell growth cell differentiation growth and shape of plants, intake of nutrition by plants, growth of forest, and interaction of plant life with environment.
- ✓ Mathematical genetics this deals with the transfer of genetic characteristics from generation to generation through the action of genes.
- ✓ Mathematical bio economics this analyzes the optimal utilization of renewable resource such as fisheries and forests.

Epidemic Models:

A simple deterministic model. In a given population at time t, let S(t) be the number of susceptible, i.e. the number those who can be infected, I(t) be the number of infected person in the population, and R(t) be the number of those removed from the population by recovery immunization, death, hospitalization or by any other means. If N(t) is the total population size, we have

$$S(t)+I(t)+R(t)=N=\text{constant}$$

We first consider simple epidemic model, i.e. models in which there are no removals. Let n be the initial number of susceptible in the population in which one infected person has been introduced so that

$$\left.\begin{aligned} S(t)+I(t)&=n+1\\ S(0)=S_0&=n\\ I(0)I_0&=1 \end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\ldots\ldots(2.2.1)$$

Now, due to infection, the number of susceptible decreases and the number of infected person increases. We assume that the rate of decrease of S(t), or the rate of increase of I(t) is proportional to the product of the number susceptible and the number of infected, so that our model gives,

$$\left.\begin{aligned} \frac{dS}{dt}&=-\beta SI\\ \frac{dI}{dt}&=\beta SI \end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\ldots\ldots(2.2.2)$$

From (2.2.1) and (2.2.2) $\dfrac{dS}{dt}=-\beta S(n=1-s)\ldots\ldots\ldots(2.2.3)$

Integrating (2.2.3) and using (2.2.1), we get

$$S(t)=\frac{n(n+1)}{n+e^{(n+1)\beta t}},\quad I(t)=\frac{(n+1)e^{(n+1)\beta t}}{n+e^{(n+1)\beta t}}\ldots\ldots\ldots(2.2.4)$$

So that $\left.\begin{matrix}\underset{t\to\infty}{Lt}\,S(t)=0\\ \underset{t\to\infty}{Lt}\,I(t)=n+1\end{matrix}\right\}\ldots\ldots\ldots\ldots\ldots\ldots(2.2.5)$

And, ultimately, all persons will be infected. In practice, public records do not show the number of infected persons; rather they show the number of new cases reported everyday. The corresponding carve gives a relation between ds/dt and t. This curve is known as the epidemic curve, and the equation for this curve is given by

$$\frac{ds}{dt}=\frac{-\beta n(n+1)\exp[(n+1)\beta t]}{[n+\exp\{n+1\}\beta t]^2}\ldots(2.2.6)$$

Which is a symmetrical unimodel with a maximum at

$$t_0=\frac{I_n n}{\beta(n+1)}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.2.7)$$

This shows that the rate of appearance of new cases rises rapidly to maximum at a time depending on β and n, and then falls to zero.

SIS model (Susceptible Infected Susceptible):

In this model, a susceptible person can become a infected at a rate proportional to SI and an infected person can recover and become susceptible again at a rate γ, so that we get the model

$$\left.\begin{matrix}\frac{dS}{dt}=-\beta SI+\gamma I\\ \frac{dI}{dt}=\beta SI-\gamma I\end{matrix}\right\}\ldots\ldots\ldots\ldots\ldots\ldots(2.2.8)$$

Which gives S(t)+I(t)=N=S(0)+I(0)=S_0+I_0,($I_0\neq 0$).....(2.2.9)

$$\frac{dI}{dt}=(\beta N-\gamma)I-\beta I^2$$

From (2.2.8) and (2.2.9 $=KI-\beta I^2$ .....(2.2.10)

Integrating (2.2.10), we get

$$I(t)=\begin{cases}\dfrac{\exp(kt)}{\beta[\exp(kt)-I/K+I_0^{-1}}, (K\neq 0)\\ \dfrac{1}{\beta t+I_0^{-1}}, (K=0)\end{cases}\ldots\ldots\ldots(.2.211)$$

$$As\ \ t\to\infty,$$

$$I(t)\to\begin{cases}N-\rho & if\ \ N>\rho=\gamma/\beta\\ 0 & if\ \ N\leq\rho=\gamma/\beta\end{cases}$$

Control of an Epidemic:

During an epidemic, an infected person can be removed from the scene of disease by quarantine and a susceptible person can be made immune by vaccination. If vaccination is performed at a rate α, then our model becomes

$$\frac{dS}{dt}=\beta S I-\alpha,\quad \frac{dI}{dt}=\beta I S-\gamma I,$$
$$\frac{dR}{dt}=\gamma,\quad \frac{dV}{dt}=\alpha,\ldots\ldots\ldots\ldots\ldots\ldots(2.3.1)$$

Where V(t) denotes the number of vaccinated persons at time t. The initial conditions

$S(0)=S_0>0,\quad I(0)=I_0>0,$

are $R(0)=V(0)=0,\ldots\ldots\ldots\ldots\ldots\ldots(2.3.2)$

Obviously, $S(t)+I(t)+R(t)+V(t)=S_0+I_0=N\ldots..(2.3.3)$

We can normalize (2.3.1) to

$$\frac{dS}{dt} = \bar{\beta} S \bar{I} - \bar{\alpha}, \qquad \frac{d\bar{I}}{dt} = \bar{\beta} \bar{I} \bar{S} - \gamma \bar{I},$$

$$\frac{d\bar{R}}{dt} = \gamma \bar{I}, \ \frac{d\bar{V}}{dt} = \bar{\alpha}, \ldots\ldots\ldots\ldots\ldots\ldots (2.3.4)$$

$$\bar{S}(0) = \bar{S}_0, \ \bar{I}(0) = \bar{I}_0, \ \bar{R}(0) = 0, \ \bar{V}(0) = 0, \ldots. (2.3.5)$$

Where $\bar{S}(t) = \frac{1}{N} S(t), \ \bar{I}(t) = \frac{1}{N} I(t), \bar{R}(t) = \frac{1}{N} R(t), \ \bar{V}(t) = \frac{1}{N} V(t), \ldots\ldots. (2.3.6)$

$$\bar{\alpha} = \frac{1}{N} \alpha, \qquad \bar{\beta} = \frac{1}{N} \beta \ \ldots\ldots\ldots\ldots\ldots (2.3.7)$$

The normalized equations can be obtained from the original equations by putting N = 1. In fact, $\bar{S}, \bar{I}, \bar{R}, \bar{V}$ are the proportions of the populations of the various categories. The control problem arises because vaccination involves costs and we have to minimize the costs for achieving some pre-assigned objectives. The cost will depend on the vaccination rate and may be taken as a nonlinear function c(α) of the rate α.

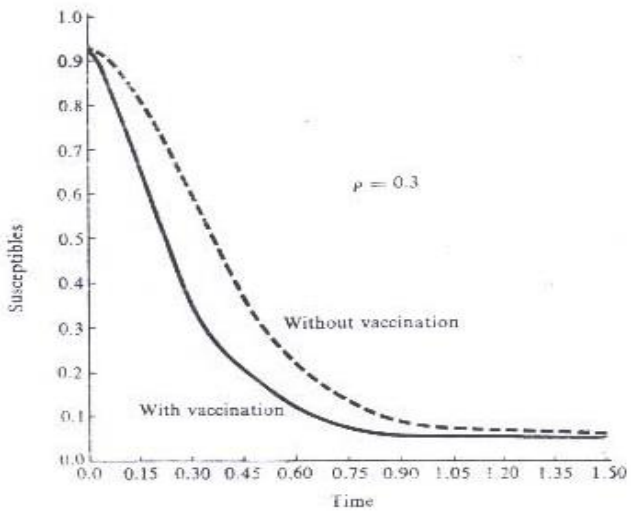
In controlling an elpidemic, we have two objectives: (i) The total proportion $\bar{R}(T) + \bar{I}(T)$ of the population affected by the epidemic over the time [0, T] is less than some prescribed number A. (ii) The maximum proportion Im(t) infected at the peak in the interval [0, T] is less than a prescribed number B. Thus our optimization problem can be stated as follows: Given a cost function c(α) and the positive constants $\bar{I}_0, \bar{S}_0, \bar{A}, \bar{B} \text{ and } \bar{T}$, we have to choose $\alpha(t)$ such that the system of equations (2.3.4)-(2.3.5) yields a solution satisfying the conditions

$$\bar{I}(T) + \bar{R}(T) \le A, \ldots\ldots\ldots\ldots\ldots (2.3.8)$$

$$\max_{[0,T]} \bar{I}(T) \le B, \ldots\ldots\ldots\ldots\ldots (2.3.9)$$

The problem has been solved by using the technique of dynamic programming. The effect of vaccination on the shape of the trajectories can be easily seen. If at any time the number of susceptible is reduced by vaccination without changing the number of infectives, the point (S, I) in phase plane moves horizontally to the left to another trajectory if after some time another vaccination redoes the number of susceptible, then the point again moves horizontally to the left to another trajectory, and so on. When there is continuous vaccination, the new trajectories cut the old trajectories. Alternatively form (2.3.1),

$$\frac{dI}{dS} = \frac{\beta I(S - \rho)}{-(\beta I S + \alpha)} \ \text{or} \ \frac{d\bar{I}}{dS} = \frac{\bar{\beta} \bar{I}(\bar{S} - \bar{\rho})}{-(\bar{\beta} \bar{S} \bar{I} + \bar{\alpha})} \ldots\ldots (2.3.10)$$



If S > ρ, dI/dS is negative for both vaccinated and non-vacinated cases, but its magnitude is less for vaccinated cases than for non-vaccinated cases. If S < ρ, dI/dS is positive for both cases, but again its magnitude is less for the vaccinated than for the non-vaccinated. The maximum infection occurs at S -ρ in both cases.

Stochastic Epidemic Model Without Removal:

Basic System of Equations:

Let $p_n(t)$ be the probability that there are n susceptible persons in the system and let $f_j(n)\Delta t + o(\Delta t)$ give the probability that the number changes to n +j in the time interval $(t, t+\Delta t)$ Here, j is any positive or negative integers, and $o(\Delta t)$ denotes an infinitesimal which is such that

$$\frac{0(\Delta t)}{\Delta t} \to 0 \, as \, \Delta t \to 0 \ldots\ldots\ldots\ldots\ldots\ldots (2.4.1)$$

The probability that there is no change in the time interval $(t, t+\Delta t)$ is then given by

$$1 - \sum_{j \neq 0} f_j(n)\Delta t + 0(\Delta t) \ldots\ldots\ldots\ldots\ldots\ldots (2.4.2)$$

Using the theorems of total and compound probabilities, we get

$$P_n(t+\Delta t) = P_n(t)[1 - \sum_{j \neq 0} f_j(n)\Delta t] + \sum_{j \neq 0} P_{n-j}(t) f_j(n-j)\Delta t + 0(\Delta t) \ldots\ldots (2.4.3)$$

So that

$$\frac{P_n(t+\Delta t) = P_n(t)}{\Delta t} = P_n(t)[1 - \sum_{j \neq 0} f_j(n) + \sum_{j \neq 0} P_{n-j}(t) f_j(n-j) \frac{0(\Delta t)}{\Delta t} + 0(\Delta t) \ldots\ldots (2.4.4)$$

Proceeding to the limit as $\Delta t \to 0,$ we obtain

$$\frac{DP_n}{Dt} = P_n \sum_{j \neq 0} f_j(n) + \sum_{j \neq 0} P_{n-j}(t) f_j(n-j) \ldots\ldots\ldots\ldots\ldots (2.4.5)$$

Multiplying (2.4.5) by x^n, summing for all n, and using the definition of the probability generating function, namely,

$$\phi(x,t) = \sum_{n=0}^{\infty} P_n(t) x^n, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.4.6)$$

We get $\frac{\partial \phi}{dt} = -\sum_{j \neq 0} \sum_{n} f_i(n) P_n X^n + \sum_{j \neq 0} \sum_{n} P_{n-j} f_j(n-j) x^{n-j} \ldots\ldots\ldots\ldots\ldots (2.4.7)$

Which gives the basic partial differential equation

$$\frac{\partial \phi}{dt} = -\sum_{j \neq 0} (x^j - 1) f_j(x \frac{\partial}{\partial x}) \phi(x,t) \ldots\ldots\ldots\ldots\ldots\ldots (2.4.8)$$

Now we make use of the relations

$$(x \frac{\partial}{\partial x}) \phi = \sum_n n P_n(t) x^n$$

$$(x \frac{\partial}{\partial x})^2 \phi = \sum_n n^2 P_n(t) x^n$$

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$$(x \frac{\partial}{\partial x}) \phi = \sum_n n^m P_n(t) x^n, m = 1,2,3, \ldots\ldots\ldots\ldots\ldots (2.4.9)$$

to get $\psi(x \frac{\partial}{\partial x}) \phi = \sum_n \psi(n) P_n(t) x^n, \ldots\ldots\ldots\ldots\ldots\ldots (2.4.10)$

Where $\psi(x)$ is any polynomial functions of x. In order to find all the probabilities, we either solve the finite system of differential-difference equations (2.4.5) or solve the partial differential equation (2.4.8) subject to the initial condition

$$\phi(x,0) = \sum_n P_n(0)x^n = x^{n_0}, \quad \text{.....(2.4.11)}$$

When n_0 is the number of susceptible in the in the system at t = 0. Similarly, for a two-dimensional stochastic process the partial differential equation corresponding to (2.4.7) is obtained as

$$\frac{\partial \phi}{dt} = \sum_{j\neq 0} \sum_{K\neq 0} (x^j y^k - 1) f_{j,k}\left(x\frac{\partial}{\partial x}, y\frac{\partial}{\partial y}\right)\phi(x,y,t), \quad \text{.....(2.4.12)}$$

$$\phi(x,y,t), = \sum_m \sum_n P_{mn}(t)x^m y^n \quad \text{.....(2.4.13)}$$

Where

With $p_{mn}(t)$ as the probability that there are m individuals of the first kind and n individuals of the second kind and $f_{j,k}(m,n)\Delta t + o(\Delta t)$ as the probability of the number of the two kinds of individuals, changing from m to m + j and from n to n + k in the time interval $(t, t+\Delta t)$

3. Economic Impact of Malaria-Related Death in Endemic Regions:

Malaria is a parasitic disease and is transmitted by anopheles mosquitoes, which breed in surface water pools where environmental conditions are suitable for both vector and parasite development. There are four species of the protozoa of genus phasmodium, which are P. Vivex, P.Ovale, P. Malarie and P. Falcilparum. The parasite P. Falciparum is the main cause of morbidity and mortality. The parasite has two hosts: human and mosquito. The female mosquito species picks up the infection after biting an human and when it bites a susceptible human, person becomes infective after some times. The world Health organization has estimated approximately 110 million clinical cases of malaria worldwide per year-over 80% of these occur in Africa south of the sahara. It is endemic in certain parts of Africa and North-Eastern part of India, where this disease is spread mainly by lethal parasite called phasmodium falciparum. It also imposes a considerable economic burden resulting from high morbichit levels within the adult population in endemic regions as economic growth of the nation depends upon the labor class of the nation. The modeling of the transmission of malaria has started in the early part of 19th century. Although there have been several investigations related to malaria in different. Contexts, the study of its impact on economic growth has not been conducted. Keeping in view of the above, a mathematical model for the spread of malaria with its impact on economic growth is proposed and analyzed by considering criss-cross interactions between female mosquitoes and human populations, when both human and mosquito population are variable.

The Model:

The total population density N under consideration is divided into two disjoint classes. First one is the children class Nc and the other class Na of rest of the populaton, which actively contribute to the development/production of the nation. Again the total population of children class Nc is divided into susceptible children Xc and the infective children Yc. Similarly the population Na is also divided into the suscepltible class Xa and the infective class Ya. It is assumed that the mosquito population Nm is growing logistically in the environment with intrinsic growth rate r_m and the carrying capacity M. Also the total mosquito population Nm is divided into the class of susceptible mosquitos X_m and the class of infective mosquitos Ym. In view of the above and by considering the criss-cross interaction of mosquito population with human population, the model can be proposed as follows.

$$\dot{X}_c = A_c + bN_a - d_c X_c - \beta_c X_c Y_m + \nu_c Y_c - \lambda X_c$$

$$\dot{Y}_c = \beta_c X_c Y_m - d_c Y_c - \nu_c Y_c - \alpha_c Y_c - \lambda Y_c$$

$$\dot{N}_c = A_c + bN_a - d_c N_c - \lambda N_c - \alpha_c Y_c$$

$$X_a = A_a + \lambda X_c - d_a X_a - \beta_a X_a y_m + V_a y_a \quad \text{.....(3.2.1)}$$

$$\dot{Y}_a = \lambda Y_c - d_a Y_a + \beta_a X_a Y_m - \nu_a Y_a - \alpha_a Y_a$$

$$\dot{N}_a = A_a + \lambda N_c - d_a N_a - \alpha_a Y_a$$

$$\dot{X}_m = \left(b_m - \frac{r_m a_m N_m}{M}\right) N_m - \left\{d_m - (1-a_m)\frac{r_m N_m}{M}\right\} X_m - \beta_{mc} X_m Y_c - \beta_{ma} X_m Y_a - \alpha_m X_m$$

$$\dot{Y}_m = \beta_{mc} X_m Y_c + \beta_{ma} X_m Y_a - \alpha_m Y_m - \left\{d_m - (1-a_m)\frac{r_m N_m}{M}\right\} Y_m$$

$$\dot{N}_m = r_m \left(1 - \frac{N_m}{M}\right) N_m - \alpha_m N_m,$$

With initial conditions $X_c(0)>0$, $Y_c(0)>0$, $N_c(0)>0$, $X_a(0)>0$, $Y_a(0)>0$, $N_a(0)>0$, $X_m(0)>0$, $Y_m(0)>0$, $N_m(0)>0$, where A_x is the rate of immigration in x class (x = C stands for children class and x = a stands for the other class of rest of the population); b is the birth rate constant; dx is the death rate constant in x class; βx is the transmission coefficient corresponding to movement of individuals from susceptible class to infective class by interacting with infective mosquitoes, in x class;x Vx is the recovery rate constant in x class; αx is the disease related death rate constant in x class; λ is the rate constant corresponding to movement of individual from children class Nc to Na; Tm = bm – dm is the intrinsic growth rate constant of mosquito population, where bm and dm are its birth and death rate constants; $0 \le am \le 1$ is convex combination constant, which regulates logistic birth and logistic death rates of mosquito population; M is the carrying capacity of mosquito population; β_{mc} and β_{ma} are the transmission coefficients corresponding to movement of susceptible mosquitoes to infective class due to interaction of infective individuals of the children class Nc and the class Na of rest of the population respectively; α_m is the death rate constant of mosquito population due to control measures. Here it is noted that Ac = 0 implies no immigration in children class Nc and Aa = 0 implies no immigration in the class Na of rest of the population. Also by taking the cobb-Douglas production function, the simple economics growth model is as follows

$$D = F(X, L)$$

$$= cK^{\theta_1}, L^{\theta_2} \quad \text{..... (3.2.2)}$$

$$\dot{K} = sD - s_0 K$$

$$L = \gamma N_a, \theta_1 + \theta_2$$

with initial conditions, K(0)> 0, D(0)> 0. where c and γ are constants; k is the capital, L is Labor and D is the production function /development function; s is a constant fraction of the output wed for enhancing capital stock and s_0 the constant fraction of the capital lost due to depreciation. From last equation of the system (3.2.1) the asymptotic value of N_m is given by

$$\lim_{t\to\infty} \sup N_m(t) = \frac{M}{r_m}\{r_m - \alpha_m\} = \bar{N}_m \ (say),$$

So it is sufficient to consider following subsystem of combined system of (3.2.1) and (3.2.2) using.

$$X_c + Y_c = N_c,\ X_a + Y_a = N_a,\ X_m + Y_m = N_m,\ L = \gamma N_a \ and \ D = cK^{\theta_1} L^{\theta_2}.$$

$$\dot{Y}_c = \beta_c (N_c - Y_c) Y_m - d_c Y_c - \nu_c Y_c - \alpha_c Y_c - \lambda Y_c$$

$$\dot{N}_c = A_c + bN_a - (d_c + \lambda)N_c - \alpha_c Y_c$$

$$\dot{Y}_a = \lambda Y_c - (d_a + \nu_a + \alpha_a)Y_a + \beta_a(N_a - Y_a)Y_m$$

$$\dot{N}_a = A_a + \lambda N_c - d_a N_a - \alpha_a Y_a$$

$$\dot{Y}_m = \beta_{mc}(\bar{N}_m - Y_m)Y_c + \beta_{ma}(\bar{N}_m - Y_m)Y_a - \alpha_m Y_m - \left\{d_m + (1-a_m)\frac{r_m \bar{N}_m}{M}\right\}Y_m$$

$$K = sck^{\theta_1}(\gamma N_a)^{\theta_2} - s_0 K \quad \text{.....(3.2.3)}$$

The continuity of right sides of (3.2.3) and its derivatives imply that unique solution exist,

Equilibrium Analysis:

For the equilibrium point of the system (3.2.3) we get the following set of relations with K > 0 by putting right hand side of system (3.2.3) to zero.

$$N_c = \frac{d_a A_c + bA_a - (d_a \alpha_c y_c + b\alpha_a y_a)}{d_a(d_c + \lambda) - b\lambda} \quad \text{.....(3.3.1)}$$

$$N_a = \frac{\lambda A_c + (d_c + \lambda)A_a - \{\lambda \alpha_c y_c + (d_c + \lambda)\alpha_a \lambda_a}{(d_c + \lambda)d_a + \lambda b} \quad \text{.....(3.3.2)}$$

$$y_m = \frac{(\beta_{mc} y_c + \beta_{ma} y_a)\bar{N}_m}{\{\beta_{mc} y_c + \beta_{ma} y_a + \alpha_m + d_m + (1 - am)\frac{r_m \bar{N}_m}{M}\}} \quad \text{.....(3.3.3)}$$

$$y_m = \frac{(d_c + v_c + \alpha_c + \lambda)y_c}{\beta_c(N_c - y_c)} \quad \text{.....(3.3.4)}$$

$$y_m = \frac{(d_c + v_c + \alpha_c)y_a - \lambda y_c}{\beta_a(N_a - y_a)} \quad \text{.....(3.3.5)}$$

$$L = r\gamma \; N_a$$

$$K^{(1-\theta_1)} = \frac{SCL^{\theta_2}}{S_0} \quad \text{.....(3.3.6)}$$

The equations (3.3.1) , (3.3.3) and (3.3.4) gives the following relation between Ya and Yc. $L_1 Y_c^2 + M_1 Y_c Y_a + N_1 Y_a^2 + O_1 Y_c + P_1 Y_a = 0$.....(3.3.7)

While the equations (3.3.2), (3.3.3) and (3.3.5) give another relation between Ya and Yc as follows $L_2 Y_c^2 + M_2 Y_c Y_a + N_2 Y_a^2 + O_2 Y_c + P_2 Y_a = 0$.....(3.3.8)

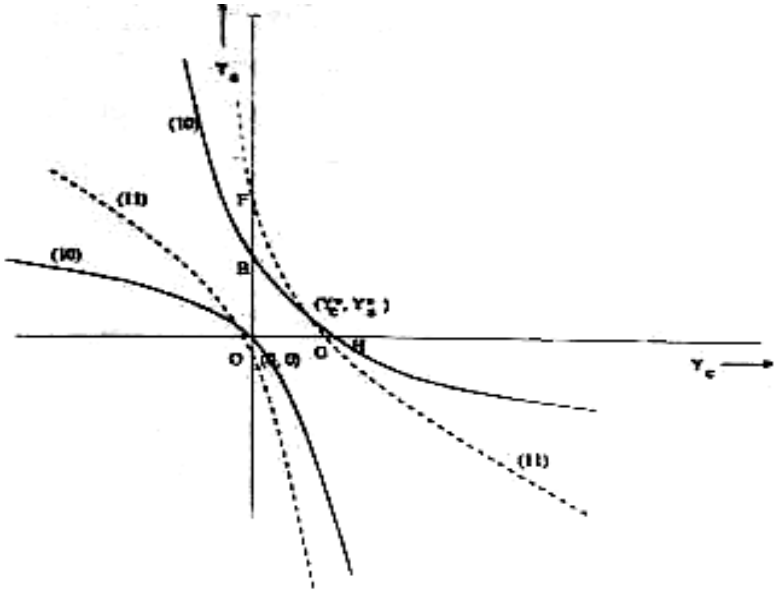

Figure 1

Where,

$$L_1 = \beta_{mc}(d_c + v_c + \alpha_c + \lambda)\{d_a(d_c + \lambda) - \lambda b\} + \beta_{mc}\beta_c \bar{N}_m \{d_a(d_c + \lambda) - \lambda b + d_a\alpha_c\},$$

$$M_1 = \beta_{mc}\bar{N}_m\beta_c b\alpha_a + \beta_{ma}\bar{N}_m\{d_a(d_c + \lambda) - \lambda b + d_a\alpha_c\}$$
$$+\beta_{ma}(d_c + v_c + \alpha_c + \lambda)\{d_a(d_c + \lambda) - \lambda b\},$$

$$N_1 = \beta_{ma}\bar{N}_m\beta_c b\alpha_a,$$

$$o_1 = H(d_c + v_c + \alpha_c + \lambda)\{d_a(d_c + \lambda) - \lambda b\} - \beta_c\bar{N}_m(d_a A_c + bA_a)\beta_{mc},$$

$$p_1 = -\beta_c\bar{N}_m\beta_{ma}(d_a A_c + bA_a),$$

$$H = \alpha_m + d_m + (1 - a_m)\frac{r_m\bar{N}_m}{M}$$

$$L_2 = \lambda\beta_{mc}\ \alpha_c\bar{N}_m - \{d_a(d_c + \lambda) - \lambda b\}$$

Existence of equilibrium point.

$$M_2 = \beta_{mc}\bar{N}_m\{(d_c + \lambda)(\alpha_a + d_a) - \lambda b\} + \beta_{ma}\bar{N}_m\lambda\alpha_c$$
$$-\beta_{ma}\lambda\{(d_c + \lambda)d_a + \lambda b\} + \beta_{mc}(d_a + v_a + \alpha_a)\{(d_c + \lambda)d_a - \lambda b\},$$

$$N_2 = -\beta_{ma}\bar{N}_m\{(d_c + \lambda)(\alpha_a + d_a) - \lambda b\}\beta_{ma}(d_a + v_a + \alpha_a)\{(d_c + \lambda)d_a - \lambda b\},$$

$$O_2 = H\lambda\{(d_c + \lambda)d_a - \lambda b\} - \bar{N}_m\{\lambda A_c + (d_c + \lambda)A_a\}\beta_{mc},$$

$$P_2 = H(d_a + v_a + \alpha_a)\{(d_c + \lambda)d_a - \lambda b\} - \bar{N}_m\{\lambda A_c + (d_c + \lambda)A_a\}\beta_{ma},$$

Now it is easy to check that (3.3.7) and (3.3.8) are hyperbola so plotting (3.3.7) and (3.3.8) by isocline method in fig-1, we get unique nontrivial intersecting point (Y_c*,y_a*) in first quadrant if

$$d_a(d_c + \lambda) > \lambda b, \quad \text{.... (3.3.9)}$$

$$\beta_c\bar{N}_m(d_a A_c + bA_a)\beta_{mc} > H(d_c + v_c + \alpha_c + \lambda)\{d_a(d_c + \lambda) - \lambda b\}, \quad \text{.... (3.3.10)}$$

$$\bar{N}_m\{XA_c + (d_c + \lambda)A_a\{\beta_{ma} - H(d_a + v_a + \alpha_a)\{d_a(d_c + \lambda) - \lambda b\}, \quad \text{..... (3.3.11)}$$

$$\alpha_c\bar{N}_m > \{d_a(d_c + \lambda) - \lambda b\}, \quad \text{..... (3.3.12)}$$

And either of the following two conditions are satisfied. (i) OE < OF and OH > OG or (ii) OE>OF and OH < OG, Where OE & OF and OH & OG are intercepts on Ya and Yc axes by carves (3.3.7) and (3.3.8) respectively and are given by

$$OE = \frac{d_a A_c + bA_a}{b\alpha_a},$$

$$OF = \frac{\bar{N}_m\{\lambda A_c + (d_c + \lambda)A_a\}\beta_{ma} - H(d_a + v_a + \alpha_a)\{(d_c + \lambda)d_a - \lambda b\}}{\beta_{ma}\bar{N}_m[\{(d_c + \lambda)d_a - \lambda b\} + (d_c + \lambda)\alpha_a] + \beta_{ma}(d_a + v_a + \alpha_a)\{d_a(d_c + \lambda)d_a - \lambda b\}},$$

$$OH = \frac{\beta_c\bar{N}_m(d_a A_c + bA_a)\beta_{mc} - H(d_c + V_c + \alpha_c + \lambda)\{(d_c + \lambda)d_a - \lambda b\}}{\beta_{mc}(d_c + V_c + \alpha_c + \lambda)\{(d_c + \lambda)d_a - \lambda b\} + \beta_{mc}\beta_c\bar{N}_m[\{(d_c + \lambda)d_a - \lambda b\} + d_a\alpha_c]}$$

$$OG = \frac{\bar{N}_m\{\lambda A_c + (d_c + \lambda)A_a\}\beta_{mc} + H\lambda\{(d_c + \lambda)d_a - \lambda b\}}{\lambda\beta_{mc}(\alpha_c\bar{N}_m - L)}$$

After finding values of Y_c* andY_a*, positive values of N_c*,N_a*,Y_m*, and K*. Are obtained from (3.3.1), (3.3.2), (3.3.3) and (3.3.6) under following conditions

$$d_a A_c + bA_a > (d_a\alpha_c y_c + b\alpha_a y_a), \quad \text{..... (3.3.13)}$$

$$\lambda A_c + (d_c + \lambda)A_a > \{\lambda\alpha_c y_c + (d_c + \lambda)\alpha_a y_a\}. \quad \text{.. (3.3.14)}$$

Thus under conditions (3.3.9) to (3.3.14), we get a nontrivial equilibrium point as E*(Y_c*,N_c*,Y_a*,N_a*,Y_m*,K*)

Linear Stability Analysis:

The variational matrix M* corresponding to the system (3) at the equilibrium E*(Y_c*,N_c*,Y_a*,N_a*,Y_m*,K*) is given by

$$M^* = \begin{pmatrix} m_{11} & \beta_c Y_m^* & 0 & 0 & \beta_c(N_c^* - Y_c^*) & 0 \\ -\alpha_c & -(d_e+\lambda) & 0 & b & 0 & 0 \\ \lambda & 0 & -(d_a+v_a+\alpha+\beta_a Y_m^*) & \beta_a Y_m^* & \beta_a(N_a^* - Y_a^*) & 0 \\ 0 & \lambda & -\alpha_a & -d_a & 0 & 0 \\ \beta_{mc}(\bar{N}_m - Y_m^*) & 0 & \beta_{ma}(\bar{N}_m^* - Y_m^*) & 0 & m_{55} & 0 \\ 0 & 0 & 0 & m_{64} & 0 & m_{66} \end{pmatrix}$$

Where, $m_{11} = -(\beta_c Y^*_m + d_c + V_c + \alpha_c + \lambda)$

$$m_{55} = -\{\beta_{mc} Y^*_c + \beta_{ma} Y^*_a + \alpha_m + d_m + (1-a_m)\frac{rm}{M}\bar{N}_m,$$

$$m_{64} = sc\theta_2 \gamma^{\theta 2} K^{*\theta 1} N_*^{\theta_2 - 1} \quad and$$

$$m_{66} = -(1-\theta_1)s_0.$$

Clearly one characteristic root of the above matrix M* is m_{66} and other characteristic roots are given by following equation

$$\psi^5 + g_4\psi^4 + g_3\psi^3 + g_2\psi^2 + g_1\psi + g_0 = 0,$$

Where g_4, g_3, g_2, g_1 and g_0 are easily derived by matrix M*. Using Routh-Hurwitz criteria. The eaquilibrium E* is locally asymptotical stable if following conditions are satisfied

$$g_4 > 0, \begin{vmatrix} g_4 & g_2 \\ 1 & g_3 \end{vmatrix} > 0, \begin{vmatrix} g_4 & g_2 & 0 \\ 1 & g_3 & g_1 \\ 0 & g_4 & g_2 \end{vmatrix} > 0, \begin{vmatrix} g_4 & g_2 & g_0 & 0 \\ 1 & g_3 & g_1 & 0 \\ 0 & g_4 & g_2 & g_0 \\ 0 & 1 & g_3 & g_1 \end{vmatrix} > 0, \begin{vmatrix} g_4 & g_2 & g_0 & 0 & 0 \\ 1 & g_3 & g_1 & 0 & 0 \\ 0 & g_4 & g_2 & g_0 & 0 \\ 0 & 1 & g_3 & g_1 & 0 \\ 0 & 0 & g_4 & g_2 & g_0 \end{vmatrix} \quad \text{.....(3.3.15)}$$

It is easy to check that first two inequalities of (3.3.15) are obions. Thus the equilibrium E* is locally stable provided the third and the fourth inequalities are satisfied as then fifth is obvious.

Simulation:

To show the stability of the equilibrium point, numerically, the system (3.2.3) is integrated using fourth order Range-Kutta method with the following set of parameters, which satisfies local stability conditions mentioned above.

$$\beta_c = 0.00000025, d_c = 0.0003, v_c = 0.0121,$$
$$\alpha_c = 0.0005, \lambda = 0.00005, A_c = 2, b = 0.00004$$
$$d_a = 0.0004, v_a = 0.0123, \alpha_a = 0.0005$$
$$\beta_a = 0.00000025, A_a = 4, \beta_{mc} = 0.00000025 = \beta_{ma}$$
$$\alpha_m = 0.025, d_m = 0.08, a_m = 0.999, Tm = 0.9,$$
$$S_0 = 0.005, S_1 = 0.0002, \theta_1 = 0.2, \theta_2 = 0.8$$

M=1000000,s=0.5, c = 0.3, γ =0.6

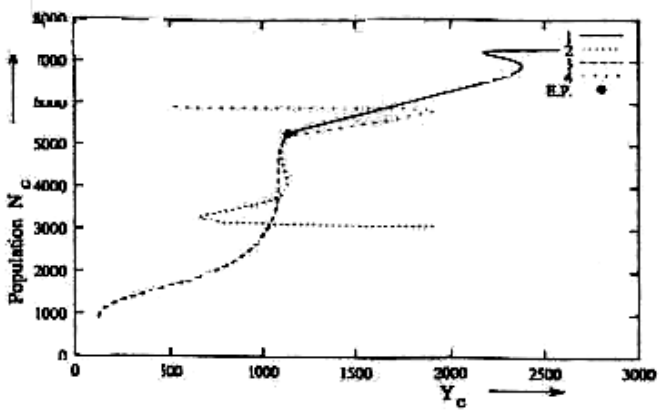

Figure 2

All parameters are in per day. The equilibrium values $E^*(Y_c^*, N_c^*, Y_a^*, N_a^*, Y_m^*, K^*)$ for above set of parameters are following

Infective Population:

Figure 2: Variation of the infective population with the total population of the class Nc Yc* = 1142.8, Nc* = 5271.4, Ya* = 2281.1, Na* =10410.1, Ym* =14339.0, K* = 438538.9. Simulation is performed for different initial starts 1, 2, 3 and 4 as shown in figs. 2 and 3. in Fig.2 and Fig 3, the infective population against the total population of respective class are plotted and from these curves, we conclude that this equilibrium is globally stable for the chosen set of parameters. Also fig4 plots of production with time for different sets of disease related death rates (keeping other parameters same) are shown, which shows production is less in the region were the most malaria – related deaths are adults as compared to the region where most malaria related deaths are among infants and young children.

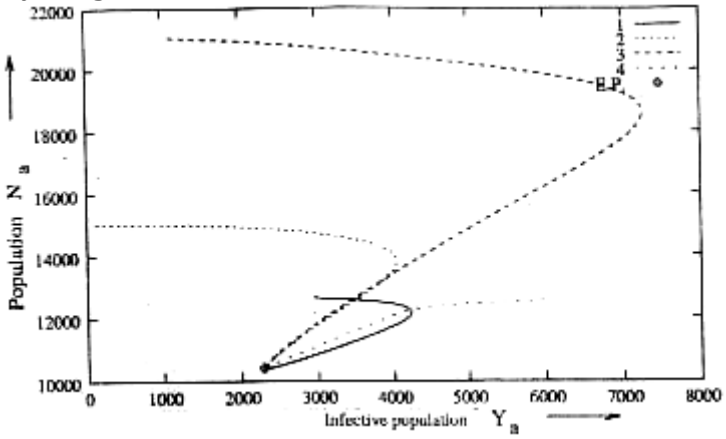

Figure 3: Variation of the infective population with the total population of the class Na.

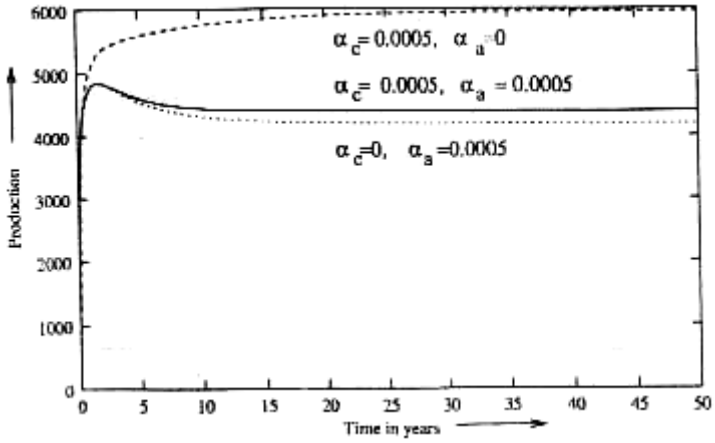

Figure 4: Variation of production with time for two different set of parameter.

4. Conclusion:

In this paper an age- dependent mathematical model is proposed and analyzed to see the effect of malaria-related death on economic growth of the region under consideration. It is shown that under certain conditions the endemic equilibrium is Locally asymptotically stable. By computer simulation it is shown that this equilibrium is in fact globally stable for the chosen set of parameters. Also by simulation, it is concluded that production is less in the region where the most malaria related deaths are among adults as compared to the region where most malaria-related deaths are among infant and young children.

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