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A BRIEF REVIEW ON INFINITE QUEUE MODEL

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Introduction:

In multiple exponential channels type one of the stations can deliver the same type of service and is equipped with same type of facilities. The element which selects one station makes this decision without any external pressure from anywhere. Due to this fact, the queue is single. The single queue (infinite length) usually breaks into smaller queues in front of each station of a single line (which has its mean rate λ) that randomly scatters itself toward three stations (S=3), each of which has an mean service rate μ. A single waiting line forms in the front of multi-service facility, within which are stationed two are more servers. Each customer is serviced by one of the server, perhaps after some waiting in the queue.

Multi-Server Model with Infinite Queue:

In this model there are S parallel servers. The arrival rate is λ and the service rate per server is μ. Because there is no limit on the number in the system, $\lambda_{eff} = \lambda$. The effect of using S parallel servers is a proportionate increase in the facility service rate. In terms of the generalized model is based on the long-run or steady-state behavior of the queuing situation which is achieved after the system has been in operation for a sufficiently long time. This type of analysis contrasts with the transient (or warm-up) behavior that prevails during the early operation of the system. One reason for not discussing the transient behavior in this chapter is its analytical complexity. Another reason is that the study of most queuing situations occurs under steady-state conditions. The generalized model assumes that both the arrival and departure rates are state dependent, meaning that they depend on the number of customers in the service facility. For example, at a highway toll booth, attendants tend to speedup toll collection during rush hours. Another example occurs in a shop with a given number of machines where the rate of breakdown decreases as the number of broken machines increases (because only working machines are capable of generating new breakdowns).

Define N = Number of customers in the system (in-queue plus in-service)

λ_n = Arrival rate given n customers in the system

μ_n = Departure rate given n customers in the system

p_n = Steady state probability of n customer in the system

The generalized model derives Pn as a function of λ_n and μ_n. These probabilities are then used to determine the system's measures of performance, such as the average queue length, the average waiting time, and the average utilization of the facility. Under steady-state conditions, for n > 0, the expected rates of floe into and out of state n must be equal. Based on the fact that state n can be changed to states n-1 and n+1 only, We get (Expected rate of flow into state n)= $\lambda_{n-1} P_{n-1} + \mu_{n+1} P_{n+1}$,

Similarly, (Expected rate of flow out of state n) = $(\lambda_n + \mu_n) P_n$, Equating the two rates, we get the following balance equation: $\lambda_{n-1} P_{n-1} + \mu_{n+1} P_{n+1} = (\lambda_n + \mu_n) P_n$,n=1,2,3,...

The balance equation associated with n=0, is $\lambda_0 P_0 = \mu_1 P_1$

For n=0, we have $P_1 = (\lambda_0 / \mu_1) P_0$. Next for n=1, we have $\lambda_0 P_0 + \mu_2 P_2 = (\lambda_1 + \mu_1) P_1$. Substituting $P_1 = (\lambda_0 / \mu_1) P_0$ and we get, $P_2 = (\lambda_1 \lambda_0 / \mu_2 \mu_1) P_0$

In general, we can show by induction that $P_n=(\lambda_{n-1}\lambda_{n-2}\ldots\lambda_0 / \mu_n \mu_{n-1}\ldots\mu_1) P_0$, n=1,2,3,... The value of p_0 is determined from the equation $\sum_{n=0}^{\infty} P_n = 1$

The M/M/S Model:

The model adopted in this work is the (M/M/S) :(∞ /FCFS) Multi-server Queuing Model .For this queuing system, it is assumed that the arrivals follow a Poisson probability distribution at an average of λ Customers per unit of time. It is also assumed that they are served on a first-come, first-served basis by any of the servers. The service times are distributed exponentially, with an average of μ customers per unit of time and number of servers S. If there are n customers in the queuing system at any point in time, then the following two cases may arise:

- ✓ If n<S (number of customers in the system is less than the number of servers), then there will be no queue. However, (S–n) number of servers will not be busy. The combined service rate will then be $\mu_n = n\mu$; $n \le S$
- ✓ If (number of customers in the system is more than or equal to the number of servers) then all servers will be busy and the maximum number of customers in the queue will be (n – S). The combined service rate will be $\mu_n = S\mu$; $n > S$

From the model the probability of having n customers in the system is given by

$$P_n = \begin{cases} \frac{\rho^n}{n!} P_0 & , \quad n \le S \\ \frac{\rho^n}{s!\, s^{(n-1)}} P_0 & , \quad n > S \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu-\lambda}\right]^{-1}$$

We now proceed to compute the performance measures of the queuing system. The expected number of the customer waiting on the queue (length of line) is given as:

$$L_q = \left\{\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\mu\lambda}{(\mu s-\lambda)^2}\right\} P_0$$

Expected number of customers in the system $L_s = L_q + \frac{\lambda}{\mu}$

Expected waiting time of customer in the queue $W_q = \frac{L_q}{\lambda}$

Average time a customer spends in the system: $W_s = \frac{L_s}{\lambda}$

Utilization factor i.e. the fractions of time servers are busy $\rho = \frac{\lambda}{\mu s}$

Where, λ = The arrival rate of customers per unit time, μ = The service rate per unit time, S = The number of servers, ρ_0 = The probability that there are no customers in the system, L_q = Expected number of customers in the queue, L_s = Expected number of customers in the system, W_q = Expected time a customer spends in the queue, W_s = Expected time a customer spend in the system.

'S' Server Model with Infinite Queue [M/M/S]:[∞/FCFS]:

This model deals with a queuing system having multi service channel, input pattern based on Poisson, service pattern based on exponential, infinite capacity and the discipline line is First Come First Served basis. We consider the situation when there is no customer in the system initially. The arrival rate and the servicing rate are one and the same for all servers. $\lambda_n = \lambda$ for all n

$$\mu_n = \begin{cases} n\mu & 1 \le n < s \\ s\mu & n \ge s \end{cases}$$

$$\rho = \lambda/\mu \text{ and } P_n = \begin{cases} \frac{\rho^n}{n!} P_0 & , \quad n \leq S \\ \frac{\rho^n}{s! s^{(n-s)}} P_0 & , \ n > S \end{cases}$$

To determine the probability of no customer in the system. We know that,

$$\sum_{n=0}^{\infty} P_n = 1$$

$$\sum_{n=0}^{s} P_n + \sum_{n=s+1}^{\infty} P_n = 1$$

$$\sum_{n=0}^{s} \frac{\rho^n}{n!} Po + \sum_{n=s+1}^{\infty} \frac{\rho^n}{s!\, s^{(n-s)}} Po = 1$$

$$Po + (\rho/1!)\, Po + (\rho^2/2!)\, Po + \ldots + (\rho^s/s!)\, Po + \frac{\rho^{s+1}}{s! s^{(1)}} Po + \frac{\rho^{s+2}}{s! s^{(2)}} Po + \frac{\rho^{s+3}}{s! s^{(3)}} Po + \ldots = 1$$

$$Po[1+(\rho/1!) + (\rho^2/2!) + \ldots + (\rho^{s-1}/(s-1)!)] + (\rho^s/s!)\, Po\, [1+(\rho/s)^1 + (\rho/s)^2 + \ldots] = 1$$

$$P_0 \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \left(1 - \frac{\rho}{s}\right)^{-1} \right] = 1$$

$$P_0 \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu - \lambda} \right] = 1$$

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu - \lambda} \right]^{-1}$$

To determine the expected number of customers in the queue, We know that

$$L_q = \sum_{n=s}^{\infty} (n-s) P_0$$

$$= \sum_{k=o}^{\infty} (k) P_k + s$$

$$= \sum_{k=0}^{\infty} k \left(\frac{\rho^{k+s}}{s^k s!}\right) Po$$

$$= \left(\frac{\rho^{1+s}}{s^1 s!}\right) P_0 \sum_{k=0}^{\infty} k \left(\frac{\rho}{s}\right)^{k-1}$$

$$= \left(\frac{\rho^{1+s}}{s^1 s!}\right) P_0 \frac{d}{d(\frac{\rho}{s})} \sum_{k=0}^{\infty} \left(\frac{\rho}{s}\right)^k$$

$$= \left(\frac{\rho^{1+s}}{s^1 s!}\right) P_0 \frac{d}{d(\frac{\rho}{s})} \left(1 - \frac{\rho}{s}\right)^{-1}$$

$$= \left(\frac{\rho^{1+s}}{s^1 s!}\right) P_0 \left(1 - \frac{\rho}{s}\right)^{-2}$$

$$= \frac{1}{(s-1)! s^2} (\rho)^{(s+1)} \frac{(s\mu)^2 \lambda}{(s\mu - \lambda)^2} P_0$$

$$= \frac{1}{(s-1)! s^2} \left(\frac{\lambda}{\mu}\right)^{(s+1)} \frac{(s\mu)^2 \lambda}{(s\mu - \lambda)^2} P_0$$

$$L_q = \left[\frac{1}{(s-1)!} \left(\frac{\lambda}{\mu}\right)^s \frac{\lambda\mu}{(s\mu - \lambda)^2} \right] P_0$$

To determine the expected number of customers in the system (Ls) can determined as follows: We know that

$$L_S = \sum_{n=0}^{\infty} n P_n = \sum_{n=0}^{s} n \frac{\rho^n}{n!} Po + \sum_{n=s+1}^{\infty} n \frac{\rho^n}{s! s^{(n-s)}} Po$$

$$= P_0 \sum_{n=0}^{s} \frac{\rho^n}{(n-1)!} + \left(\frac{Po}{s!}\right) \sum_{n=s+1}^{\infty} n \frac{\rho^n}{s^{(n-s)}} = P_0\, \rho \sum_{n=0}^{s-1} \frac{\rho^n}{(n)!} + \left(\frac{Po}{s!}\right) \rho^s \sum_{n=1}^{\infty} (s+n) \frac{\rho^n}{s^n}$$

$$= P_0\, \rho \sum_{n=0}^{s-1} \frac{\rho^n}{(n)!} + \left(\frac{Po}{s!}\right) \rho^s \left[\sum_{n=1}^{\infty} s \frac{\rho^n}{s^n} + \sum_{n=1}^{\infty} n \frac{\rho^n}{s^n} \right]$$

$$= P_0\,\rho\,\sum_{n=0}^{s-1}\frac{\rho^n}{(n)!} + \left(\frac{Po}{s!}\right)\rho^s\left[s\left(\frac{\rho}{s}\right)\sum_{n=0}^{\infty}\frac{\rho^n}{s^n} + \left(\frac{\rho}{s}\right)\sum_{n=1}^{\infty} n\left(\frac{\rho}{s}\right)^{n-1}\right]$$

$$=P_0\,\rho\,\sum_{n=0}^{s-1}\frac{\rho^n}{(n)!} + \left(\frac{Po}{s!}\right)\rho^s\; s\left(\frac{\rho}{s}\right)\sum_{n=0}^{\infty}\frac{\rho^n}{s^n} + \left(\frac{\rho^s Po}{s!}\right)\left(\frac{\rho}{s}\right)\sum_{n=1}^{\infty} n\left(\frac{\rho}{s}\right)^{n-1}$$

$$= \rho\, P_0\sum_{n=0}^{s-1}\frac{\rho^n}{(n)!} + \left(\frac{Po}{s!}\right)\rho^{s+1}\sum_{n=0}^{\infty}\left(\frac{\rho}{s}\right)^n + \left(\frac{\rho^{s+1}Po}{s\,s!}\right)\sum_{n=1}^{\infty}\frac{d}{d\left(\frac{\rho}{s}\right)}\left(\frac{\rho}{s}\right)^n$$

$$=\rho\, P_0\,\sum_{n=0}^{s-1}\frac{\rho^n}{n!} + \left(\frac{Po}{s!}\right)\rho^{s+1}\left(1-\frac{\rho}{s}\right)^{-1} + \left(\frac{\rho^{s+1}Po}{s\,s!}\right)\frac{d}{d\left(\frac{\rho}{s}\right)}\sum_{n=1}^{\infty}\left(\frac{\rho}{s}\right)^n$$

$$= \rho Po\,\sum_{n=0}^{s-1}\frac{\rho^n}{n!} + \left(\frac{Po}{s!}\right)\rho^{s+1}\left(\frac{1}{\left(1-\frac{\rho}{s}\right)}\right) + \left(\frac{\rho^{s+1}Po}{s\,s!}\right)\frac{d}{d\left(\frac{\rho}{s}\right)}\left(1-\frac{\rho}{s}\right)^{-1}$$

$$L_S = \rho\, Po\,\sum_{n=0}^{s-1}\frac{\rho^n}{n!} + \left(\frac{Po}{s!}\right)\rho^{s+1}\left(\frac{s}{(s-\rho)}\right) + \left(\frac{\rho^{s+1}Po}{s\,s!}\right)\left(1-\frac{\rho}{s}\right)^{-2}$$

$$L_S = \rho\, Po\,\sum_{n=0}^{s-1}\frac{\rho^n}{n!} + \left(\frac{Po}{s!}\right)\rho^{s+1}\left(\frac{s}{(s-\rho)}\right) + \left(\frac{\rho^{s+1}Po}{s!}\right)\left(\frac{s}{(s-\rho)^2}\right)$$

$$= \rho\, P_0\left[\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s\frac{s\mu}{s\mu-\lambda}\right]+) + \left(\frac{\rho^{s+1}Po}{(s-1)!}\right)\left(\frac{1}{(s-\rho)^2}\right)$$

$$= \rho\, P_0(1/\,Po) + \left(\frac{\rho^s Po}{(s-1)!}\right)\left(\frac{\rho}{(s-\rho)^2}\right)$$

$$=\rho + \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s\frac{\lambda\mu}{(s\mu-\lambda)^2}\right]P_0$$

$$L_S = \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s\frac{\lambda\mu}{(s\mu-\lambda)^2}\right]P_0 + (\lambda/\mu)$$

To determine the Expected waiting time of customers in the queue (Wq) can determined as follows: $W_q = (1/\lambda)\, L_q$

We know that $L_q = \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s\frac{\lambda\mu}{(s\mu-\lambda)^2}\right]P_0$

$$W_q = \frac{1}{\lambda}\left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s\frac{\lambda\mu}{(s\mu-\lambda)^2}\right]P_0$$

$$W_q = \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s\frac{\mu}{(s\mu-\lambda)^2}\right]P_0$$

To determine the Expected waiting time of customers in the system (Ws) can determined as follows: $W_s = (1/\lambda)\, L_s$

We know that $L_s = \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s\frac{\lambda\mu}{(s\mu-\lambda)^2}\right]P_0 + (\lambda/\mu)$

$$Ws = \frac{1}{\lambda}\left\{\left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s\frac{\lambda\mu}{(s\mu-\lambda)^2}\right]Po + (\lambda/\mu)\right\}$$

$$= \left\{\frac{1}{\lambda}\left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s\frac{\lambda\mu}{(s\mu-\lambda)^2}\right]Po + \frac{1}{\lambda}(\lambda/\mu)\right\}$$

$$Ws = \left\{\left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s\frac{\mu}{(s\mu-\lambda)^2}\right]Po + (1/\mu)\right\}$$

Utilization factor that is the fraction of time server are busy $\rho = \left(\frac{\lambda}{\mu s}\right)$

Example:

Data for this study were collected from Riverside Specialist Clinic of federal Medical Centre Chennai. The methods employed during data collection were direct observation and personal interview and questionnaire administering by the researcher. Data were collected for (4) weeks. The following assumptions were made for queuing system at the Riverside Specialist clinic which is in accordance with the queue theory.

We use TORA software to compute the performance measures of the multi-server queuing system at the Riverside Specialist hospital using arrival rate (λ)=52 patients/hr, Service rate (μ) = 6 patients/hr and number of servers (S) = 10.

Solution:

Introducing Costs into the Model, In order to evaluate and determine the optimum number of servers in the system, two opposing costs must be considered in making these decisions: (i) Service costs (ii) Waiting time costs of customers. Economic analysis of these costs helps the management to make a trade-off between the increased costs of providing better service and the decreased waiting time costs of customers derived from providing that service.

Expected Service Cost E (SC) = SCS, Where, S= number of servers, C_s=service cost of each server

Expected Waiting Costs in the System E (WC) = (λ W_s) C_w, Where λ =number of arrivals, W_s =Average time an arrival spends in the system, C_w = Opportunity cost of waiting by customers (patients)

Adding above equations we have,

$$\text{Expected Total Costs E (TC)} = \text{E (SC)} + \text{E (WC)}$$

$$\text{Expected Total Costs E (TC)} = SC_s + (\lambda W_s) C_w$$

Given data, C_s = 500, C_w = 750, S = 10, 11, 12, 13, 14, λ = 52 and μ = 6

Analysis of Data:

Table1: Performance Measures of Multi server Queuing Model at the Riverside Specialist Hospital

Scenario	S	Lambda	Mu	L'da eff	P_0	L_s	L_q	W_s	W_q
1	10	52.000	6.000	52.000	0.00012	12.39855	3.73188	0.23843	0.07177
2	11	52.000	6.000	52.000	0.00015	10.00902	1.34235	0.19248	0.02581
3	12	52.000	6.000	52.000	0.00016	9.23330	0.56663	0.17756	0.01090
4	13	52.000	6.000	52.000	0.00017	8.91794	0.25128	0.17150	0.00483
5	14	52.000	6.000	52.000	0.00017	8.77902	0.11235	0.16883	0.00216

TORA Optimization System, Windows®-version 1.00
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Monday, February 25, 2013 23:21

QUEUEING OUTPUT ANALYSIS

Title: multi-server with infinite queue
Comparative Analysis

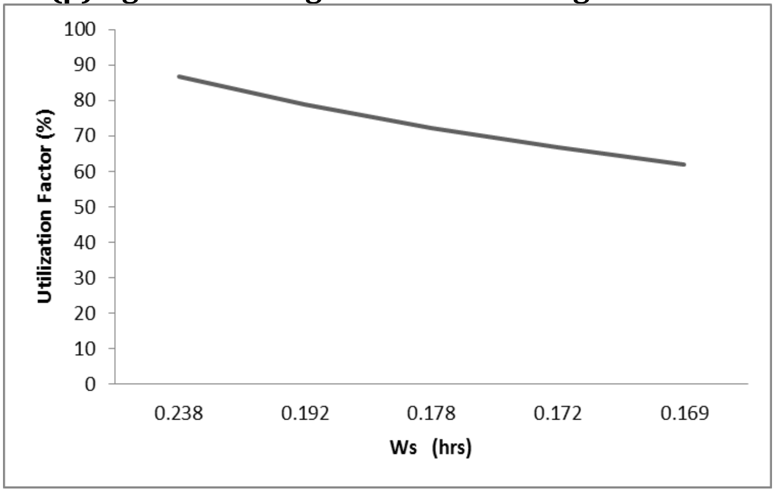
Scenario	c	Lambda	Mu	L'da eff	p0	Ls	Lq	Ws	Wq
1	10	52.00000	6.00000	52.00000	0.00012	12.39855	3.73188	0.23843	0.07177
2	11	52.00000	6.00000	52.00000	0.00015	10.00902	1.34235	0.19248	0.02581
3	12	52.00000	6.00000	52.00000	0.00016	9.23330	0.56663	0.17756	0.01090
4	13	52.00000	6.00000	52.00000	0.00017	8.91794	0.25128	0.17150	0.00483
5	14	52.00000	6.00000	52.00000	0.00017	8.77902	0.11235	0.16883	0.00216

Table 2: Summary analysis of the Multi-Server queuing Model of the Riverside Specialist Hospital

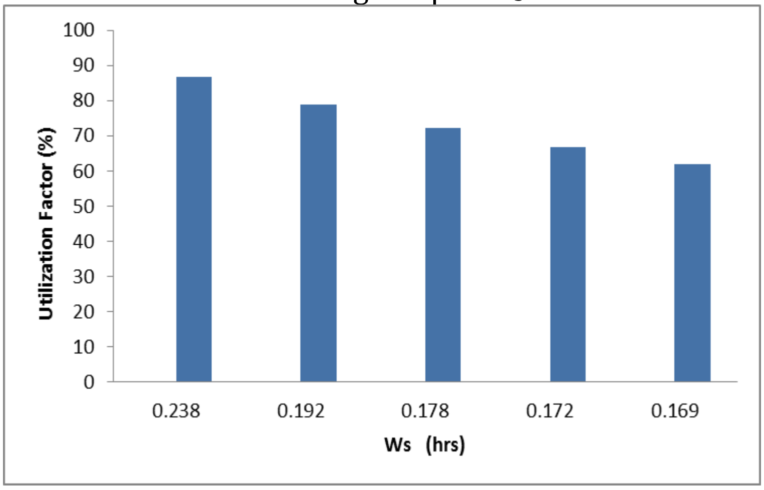
Performance measure	10 Doctors	11 Doctors	12 Doctors	13 Doctors	14 Doctors
Arrival rate (λ)	52	52	52	52	52
Service rate (μ)	6	6	6	6	6
System utilization	86.6%	78.8%	72.2%	66.7%	61.9%

L_s	12.399	10.009	9.233	8.918	8.779
L_q	3.732	1.342	0.567	0.251	0.112
W_s	0.238	0.192	0.178	0.172	0.169
W_q	0.072	0.026	0.011	0.005	0.002
P_0	0.012%	0.015%	0.016%	0.017%	0.017%
Total system cost/hr	₹ 14,282	₹ 12,988	₹ 12,942	₹ 13,208	₹ 13,591

Utilization Factor (ρ) Against Average Patients Waiting Time in the System (W_s):

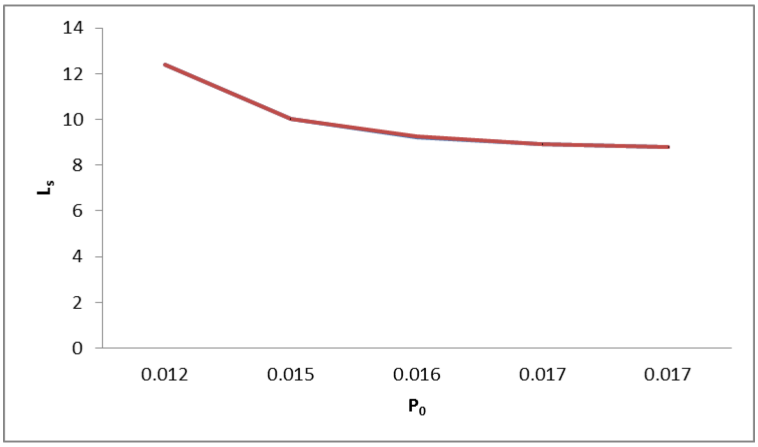


Line Diagram ρ & W_S

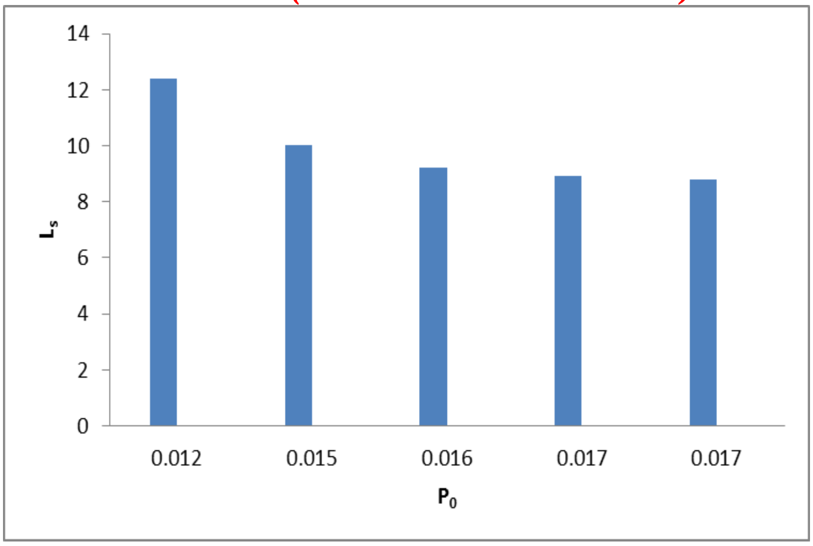


Bar Diagram (ρ) & (W_s)

Average Number of Patients in the System (L_s) against Probability of the System Being Idle (P_0):

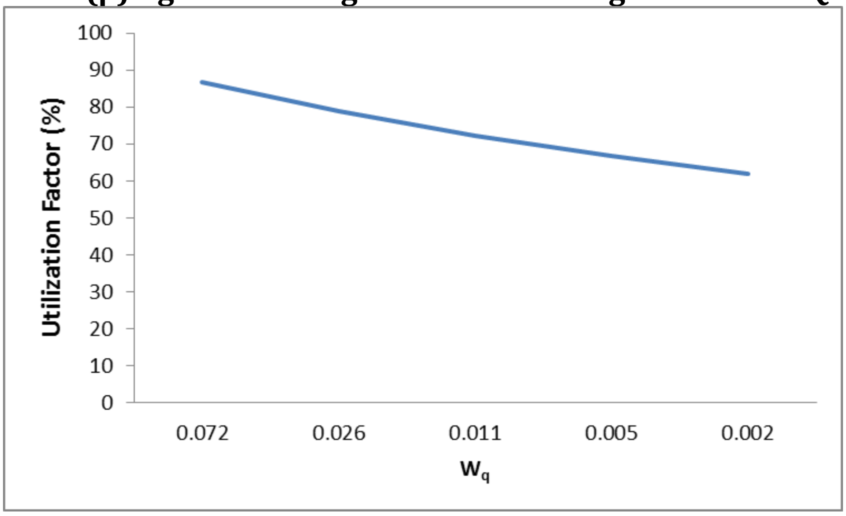


Line Diagram L_S & P_0

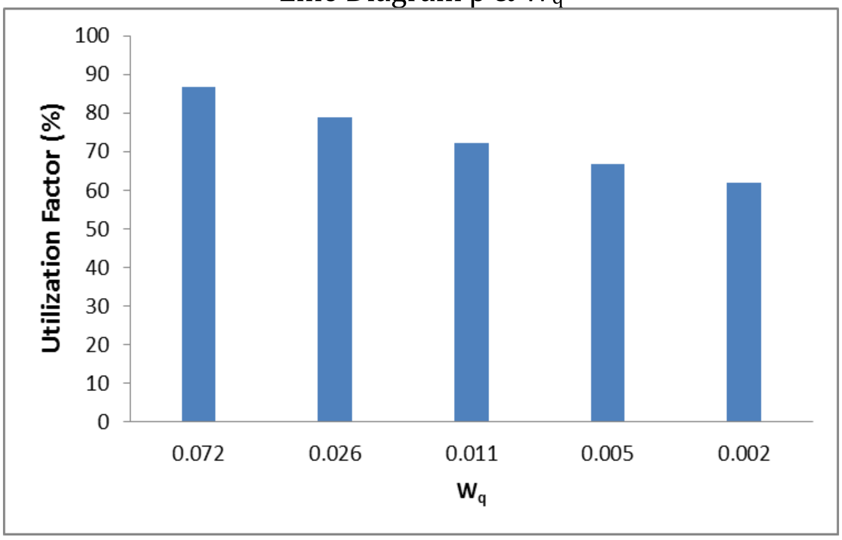


Bar Diagram L_S & P_0

Utilization Factor (ρ) Against Average Patients Waiting Time in the Queue (W_q):

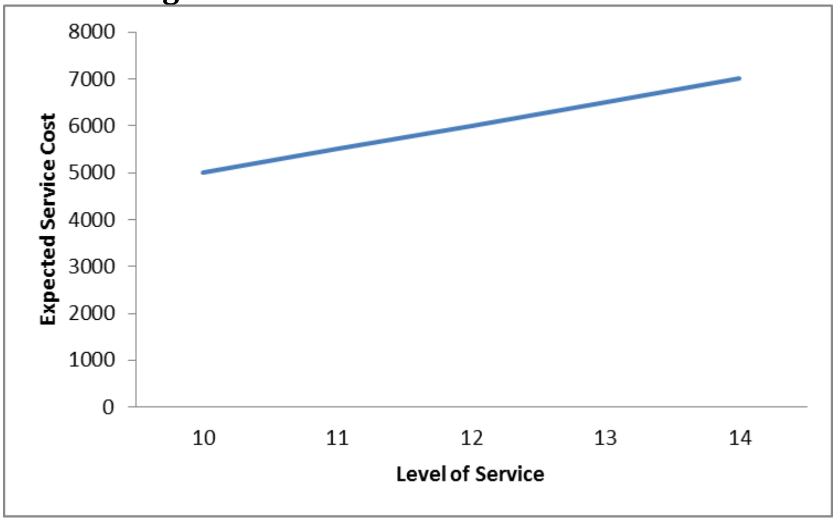


Line Diagram ρ & W_q

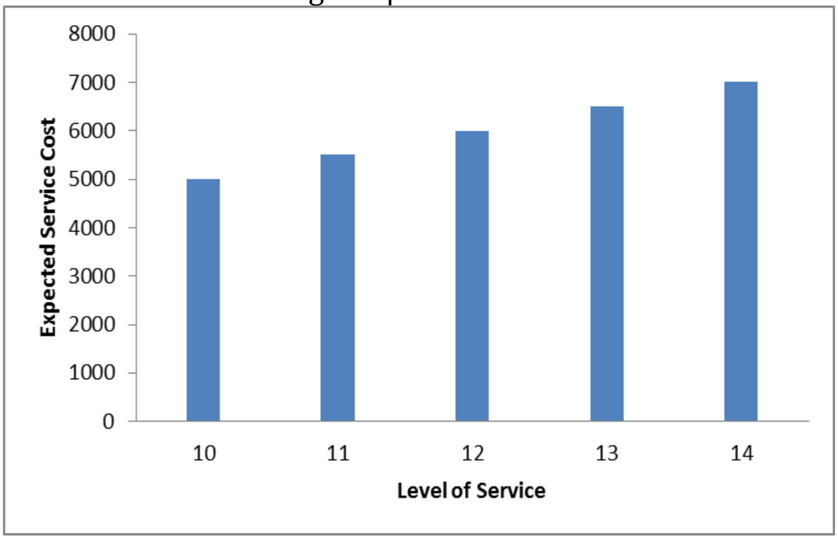


Bar Diagram (ρ) & (W_q)

Expected Service Cost Against Level of Service:

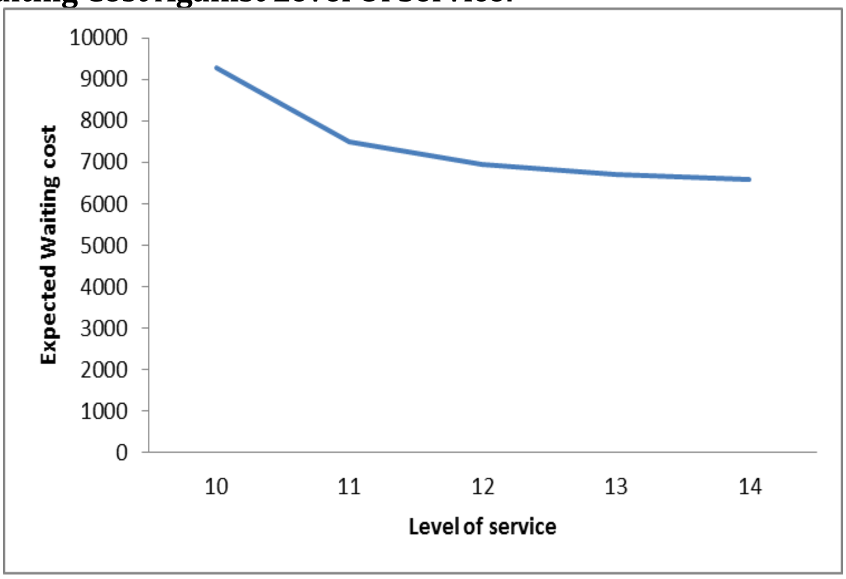


Line Diagram ρ & level of service

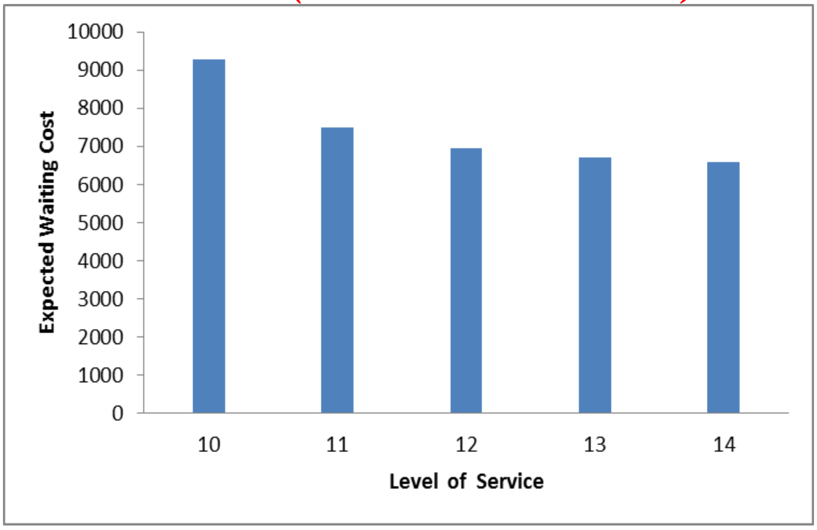


Bar Diagram (ρ) & level of service

Expected Waiting Cost Against Level Of Service:

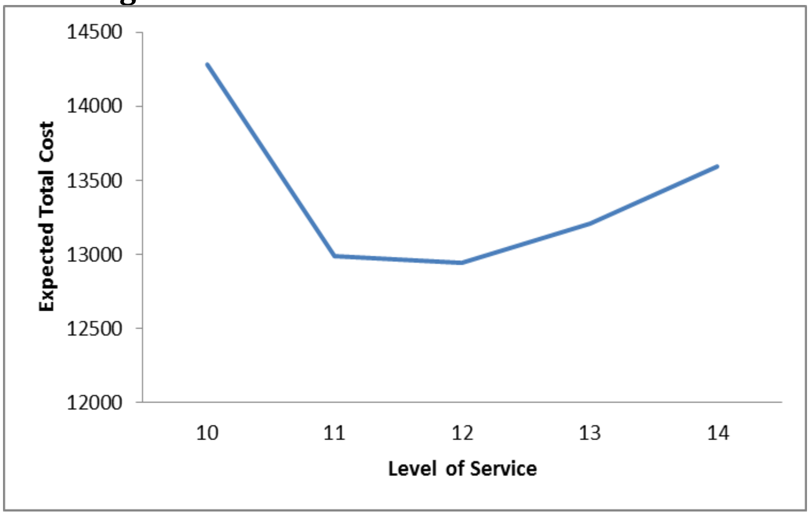


Line Diagram ρ & level of service

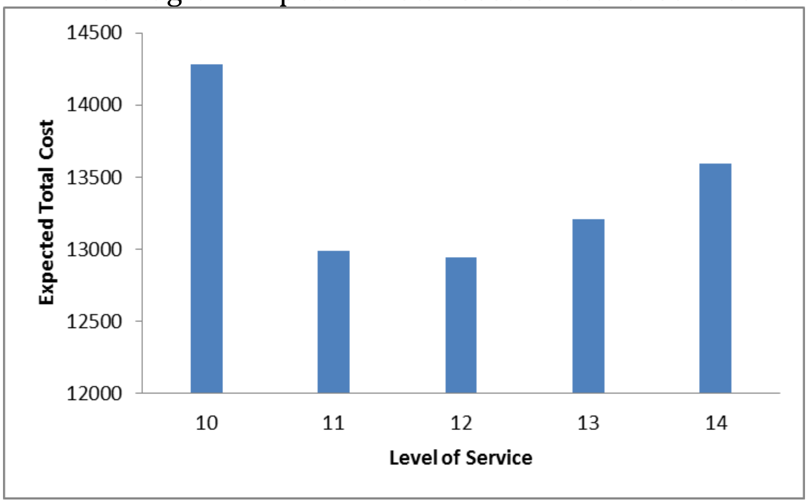


Bar Diagram (ρ) & level of service

Expected Total Cost Against Level of Service:



Line Diagram Expected Total Cost & level of service



Bar Diagram Expected Total Cost & level of service

Discussion of Result:

The graphs show that optimal server level at the Clinic is achieved when the number of servers (doctors) is 12 with a minimum total cost of 12,942 per hr as against the present server level of 10 doctors at the Clinic which have high total cost of 14,282 per hr. It should also be noted that patients' average wait time and congestion in the system is also less at this optimal server level.

Conclusion:

The queuing characteristics at the Riverside Specialist Clinic of the Federal Medical Centre , Chennai was analyzed using a Multi-server queuing Model and the Waiting and service Costs determined with a view to determining the optimal service level. The results of the analysis showed that average queue length, waiting time of patients as well as overutilization of doctors could be reduced when the service capacity level of doctors at the Clinic is increased from ten to twelve at a minimum total costs which include waiting and service costs. The operation managers can recognize the trade-off that must take place between the cost of providing good service and the cost of customers waiting time. Service cost increases as a firm attempts to raise its level of service. As service improves, the cost of time spent waiting on the line decreases. This could be done by expanding the service facilities or using models that consider cost optimization.

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